

CANONICAL CORRELATION ANALYSIS IN THE CASE OF MULTIVARIATE REPEATED MEASURES DATA

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Eigenvalues of the matrix \hat{A}

```
## [1] 1.480886e-01 4.387283e-02 3.284018e-02 2.461321e-02 1.767020e-02
## [6] 1.434321e-02 6.353271e-05
```

Canonical correlation coefficients

```
## [1] 0.384822877 0.209458413 0.181218601 0.156885966 0.132929286 0.119763148
## [7] 0.007970741
```

Eigenvectors of the matrix \hat{A}

```
##           [,1]           [,2]           [,3]           [,4]           [,5]
## [1,] 0.070344638 0.05640466 -0.027432326 0.003662585 0.13792520
## [2,] 0.015913953 0.04510670 0.005444428 -0.004455300 -0.01403631
## [3,] 0.001478003 -0.16399517 0.037055750 0.001024880 0.03116303
## [4,] -0.102662674 0.18482792 0.334347217 0.469459243 -0.42699657
## [5,] 0.681857823 -0.95312909 -0.793148047 -0.870972843 0.80336714
## [6,] 0.085729032 -0.11402983 -0.088351907 0.007864375 -0.07615806
## [7,] 0.715525689 0.11077118 0.499166057 -0.144631128 -0.38247040
##           [,6]           [,7]
## [1,] -0.004521074 0.008555062
## [2,] -0.012950439 -0.005519704
## [3,] -0.006811178 -0.004434381
## [4,] -0.058578113 0.119784695
## [5,] 0.992926424 -0.985865082
## [6,] -0.003185309 0.008220298
## [7,] 0.102083440 0.116321854
```

Eigenvalues of the matrix \hat{B}

```
## [1] 1.480886e-01 4.387283e-02 3.284018e-02 2.461321e-02 1.767020e-02
## [6] 1.434321e-02 6.353271e-05 -1.989090e-18
```

Eigenvectors of the matrix \hat{B}

```
##           [,1]           [,2]           [,3]           [,4]           [,5]
## [1,] -0.5125858380 -0.2076082876 0.1923523004 0.1080356180 2.954709e-01
## [2,] -0.0164325999 0.0178029277 0.0784506684 -0.0230910244 1.124679e-01
## [3,] 0.0008367306 0.0001591122 0.0003635302 -0.0004363356 5.041265e-05
```

```

## [4,] -0.4536054489  0.4884022383  0.3444122525 -0.5500446386 -4.824024e-01
## [5,]  0.0428364900 -0.2986878467 -0.1726513469 -0.2815581882 -2.534119e-01
## [6,] -0.0445105339 -0.7582672793  0.2534265347 -0.2278180649  1.239609e-01
## [7,]  0.5254624887  0.2089294294 -0.7506143048  0.2628240844  4.246246e-01
## [8,]  0.5012984595 -0.1010439502  0.4251715214  0.6964145544 -6.383149e-01
##      [,6]      [,7]      [,8]
## [1,]  0.3694142095  2.668742e-01 -0.3546953725
## [2,] -0.0218247012  1.561047e-01  0.1819774687
## [3,]  0.0002221692  7.751851e-05 -0.0005053137
## [4,]  0.3383635372 -3.655664e-01  0.4320586537
## [5,]  0.0425338349  3.909956e-01  0.1662428851
## [6,] -0.1115792369 -7.477333e-01  0.4747229594
## [7,]  0.8399711354 -1.666050e-01  0.4829440183
## [8,]  0.1695769148  1.761557e-01  0.4100816360

```

Canonical variables U

```

##      [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,]  0.8515613  0.76492021  0.86148180  0.87933563  0.88119192  0.89161638
## [2,] -0.2473862 -0.25710131 -0.35332085 -0.35703585 -0.36937074 -0.34611858
## [3,]  0.2269259  0.27608241  0.23079651  0.18812826  0.12792401  0.18372170
## [4,] -1.1288506 -1.20084640 -1.27178165 -1.29741098 -1.23548487 -1.21093184
## [5,]  0.2617051  0.28106957  0.27113885  0.35703716  0.39054087  0.41851909
## [6,]  1.0556672  1.07659538  1.04366478  1.00093933  0.94572902  0.87848946
## [7,]  2.3588592  2.49688080  2.45016363  2.47701022  2.39283726  2.47744456
## [8,] -0.5790071 -0.67284002 -0.76098187 -0.78627058 -0.77048919 -0.85036329
## [9,] -0.9982991 -0.98393398 -1.04375415 -1.02577451 -1.03152242 -1.04825814
## [10,] -0.2176416 -0.09571466 -0.09751121 -0.11385342 -0.11611426 -0.10138365
## [11,]  0.4223940  0.32664466  0.31815777  0.26044443  0.24493754  0.24967496
## [12,] -0.3840696 -0.40382308 -0.36579400 -0.35470927 -0.31678513 -0.37527939
## [13,] -0.6283752 -0.65035549 -0.59430991 -0.64846695 -0.59024873 -0.69524682
## [14,] -1.0847760 -1.04820781 -0.87164692 -0.84572433 -0.83889393 -0.75348043
## [15,]  0.2040664  0.22370506  0.24210621  0.23175503  0.24279904  0.23875883
## [16,] -0.1127736 -0.13307533 -0.05840898  0.03459583  0.04294963  0.04283715
##      [,7]      [,8]      [,9]      [,10]      [,11]      [,12]
## [1,]  0.84893741  0.79318598  0.7537197  0.70541494  0.72642759  0.58519887
## [2,] -0.40073205 -0.40359017 -0.3912102 -0.38857742 -0.42980147 -0.46011054
## [3,]  0.11262781  0.25265715  0.3086349  0.28713367  0.34531170  0.45152631
## [4,] -1.05579640 -1.04665381 -0.9884099 -0.87322602 -0.86170724 -0.93068944
## [5,]  0.36357993  0.25953569  0.1894485  0.06600526  0.01434484  0.05763276
## [6,]  0.78014364  0.75530031  0.7275460  0.69536371  0.68344855  0.69679297
## [7,]  2.40392008  2.43076186  2.3606437  2.26141068  2.24593477  2.21676395
## [8,] -0.81523226 -0.80848551 -0.8654196 -0.85911625 -0.83818226 -0.67113611
## [9,] -0.89920306 -0.92169470 -0.9389088 -0.92439641 -0.90045508 -0.81202932
## [10,] -0.13760631 -0.08638471 -0.1669675 -0.18702113 -0.30812038 -0.36047263
## [11,]  0.18921657  0.17871307  0.1542091  0.10235325  0.12248511  0.01948668
## [12,] -0.34903563 -0.39460545 -0.2796203 -0.19475956 -0.21438455 -0.25677182
## [13,] -0.62894846 -0.55135315 -0.4971969 -0.52119162 -0.43548553 -0.42605218
## [14,] -0.73724455 -0.77841151 -0.7650936 -0.66968079 -0.60179614 -0.49742953
## [15,]  0.27386454  0.30699254  0.2779508  0.42909926  0.32595979  0.26879406
## [16,]  0.05150875  0.01403242  0.1206742  0.07118844  0.12602029  0.11849599
##      [,13]
## [1,]  0.51479684
## [2,] -0.51666257

```

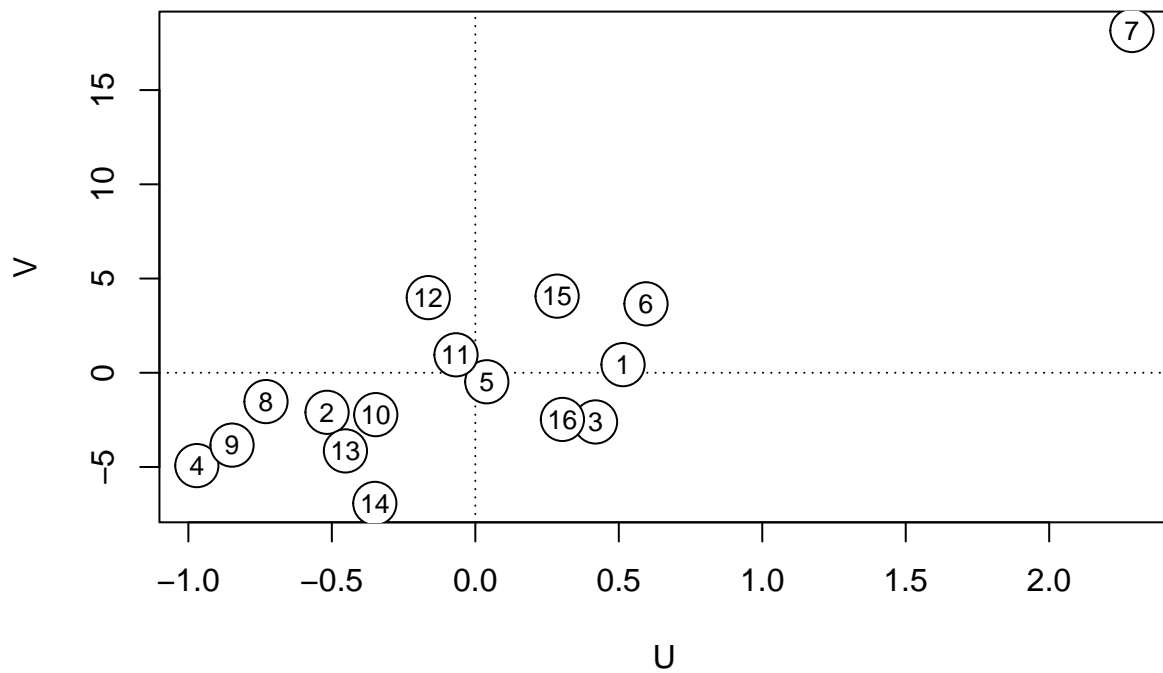
```
## [3,] 0.41894251
## [4,] -0.97048291
## [5,] 0.03999615
## [6,] 0.59481663
## [7,] 2.28836297
## [8,] -0.72945771
## [9,] -0.84776487
## [10,] -0.34698407
## [11,] -0.06731738
## [12,] -0.16402204
## [13,] -0.45262486
## [14,] -0.35018398
## [15,] 0.28502713
## [16,] 0.30355817
```

Canonical variables V

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 10.402683  9.5248795  9.0420767  8.9267972  7.2286238  5.958308
## [2,] -5.353075 -5.0280101 -5.4443552 -5.6100038 -4.8497453 -4.575747
## [3,] -8.503324 -7.5052226 -7.3204282 -7.4094378 -6.8645109 -6.701316
## [4,] -4.079317 -5.3496738 -5.4538421 -5.0832738 -4.8064676 -4.710331
## [5,] 0.243219  0.4251464  0.6450124  0.4211991  0.9722712  0.473372
## [6,] 3.814732  3.3473112  3.0951080  2.6715852  1.9964759  1.810455
## [7,] 31.651962 31.2982667 30.0229366 29.1472779 27.6631135 25.146392
## [8,] -4.740212 -4.1579047 -4.6051970 -2.4773672 -3.1011595 -1.748655
## [9,] -8.612469 -8.1399512 -8.9794889 -9.1514804 -8.0760558 -9.083232
## [10,] -8.478200 -7.4097301 -7.1659595 -6.5724801 -5.9394095 -5.422107
## [11,] 4.657162  4.6747804  5.0358661  4.1026487  3.8841528  4.053835
## [12,] 6.117633  6.5916084  7.1457717  6.8839325  6.9074457  5.833115
## [13,] -8.835789 -9.4644698 -8.0644739 -8.1810177 -7.0953432 -5.094157
## [14,] -11.629917 -11.9461097 -10.6929344 -10.3005973 -9.6973447 -9.353244
## [15,] 8.566766  8.4974274  7.8631800  7.9954532  7.2813306  7.128507
## [16,] -5.221855 -5.3583482 -5.1232723 -5.3632356 -5.5033769 -3.715195
##           [,7]      [,8]      [,9]      [,10]      [,11]      [,12]
## [1,] 4.9936618 5.5569480 3.7516264 2.411263 1.5899799 1.0683700
## [2,] -2.8901451 -2.9276396 -2.6109087 -2.585232 -2.2707513 -2.2584032
## [3,] -7.2424606 -6.4371906 -5.6282405 -4.827096 -4.1830182 -3.4093267
## [4,] -2.9281586 -2.4097057 -3.0011831 -2.753658 -4.3817180 -5.4562500
## [5,] -0.1138317 0.2451931 0.7859607 0.627466 0.9373602 0.2627267
## [6,] 1.7779888 1.5628493 2.7262698 3.329722 3.1913808 4.2101358
## [7,] 22.4702938 22.5885578 20.4663639 19.257892 18.4960562 18.0667145
## [8,] -1.6222734 -1.5108333 -2.8327450 -1.850918 -0.2607221 -2.0224312
## [9,] -8.6145545 -8.0349533 -6.9445040 -6.044545 -5.9175763 -4.5287336
## [10,] -5.3372204 -4.4135887 -4.1088899 -2.812400 -2.7318399 -1.2729195
## [11,] 2.4586601 3.0818334 3.6098621 2.696945 1.3577091 1.1467221
## [12,] 5.6323324 4.9340023 4.9301386 5.191688 5.4730354 4.7214133
## [13,] -5.3135822 -6.0473222 -5.7714809 -5.953115 -5.1709221 -3.6920890
## [14,] -8.2784166 -8.4686880 -7.7762144 -8.499965 -7.4709115 -7.4712885
## [15,] 6.4261139 5.7349862 5.4216641 5.786580 5.1343584 4.6409522
## [16,] -1.4184080 -3.4544489 -3.0177191 -3.974625 -3.7924207 -4.0055928
##           [,13]
## [1,] 0.4315625
## [2,] -2.0985830
```

```
## [3,] -2.6098810
## [4,] -4.9260722
## [5,] -0.4786050
## [6,]  3.6579888
## [7,] 18.1606869
## [8,] -1.5289058
## [9,] -3.8388828
## [10,] -2.2212480
## [11,]  0.9517610
## [12,]  3.9872909
## [13,] -4.1455243
## [14,] -6.9317847
## [15,]  4.0670695
## [16,] -2.4768729
```

The location of the various provinces in 2002



The location of the various provinces in 2014

