

# Spatio-temporal principal component analysis

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Polish regions:

1. DOLNOŚLĄSKIE
2. KUJAWSKO-POMORSKIE
3. LUBELSKIE
4. LUBUSKIE
5. ŁÓDZKIE
6. MAŁOPOLSKIE
7. MAZOWIECKIE
8. OPOLSKIE
9. PODKARPACKIE
10. PODLASKIE
11. POMORSKIE
12. ŚLĄSKIE
13. ŚWIĘTOKRZYSKIE
14. WARMIŃSKO-MAZURSKIE
15. WIELKOPOLSKIE
16. ZACHODNIOPOMORSKIE

List of variables:

1. Population per 1 km<sup>2</sup>
2. Students per 10,000 inhabitants
3. Libraries per 1,000 inhabitants
4. Production sales of industry, total per capita
5. Retail sales of goods per capita
6. Forestry and logging
7. Regional income
8. Regional expenditure
9. Targeted grants received from the state budget for indigenous tasks per km<sup>2</sup>
10. Fees and impact on the fund for environmental protection and water management
11. Communal waste-water treatment plants
12. Devastated and degraded land, remediated and developed

`## Ładowanie wymaganego pakietu: splines`

`## Ładowanie wymaganego pakietu: fds`

`## Ładowanie wymaganego pakietu: rainbow`

`## Ładowanie wymaganego pakietu: MASS`

`## Ładowanie wymaganego pakietu: pcaPP`

```

## Ładowanie wymaganego pakietu: RCurl

## Ładowanie wymaganego pakietu: deSolve

##
## Dołączanie pakietu: 'fda'

## Następujący obiekt został zakryty z 'package:graphics':
##
##      matplotlib

## Linking to GEOS 3.9.1, GDAL 3.4.3, PROJ 7.2.1; sf_use_s2() is TRUE

##
## Dołączanie pakietu: 'dplyr'

## Następujący obiekt został zakryty z 'package:MASS':
##
##      select

## Następujące obiekty zostały zakryte z 'package:stats':
##
##      filter, lag

## Następujące obiekty zostały zakryte z 'package:base':
##
##      intersect, setdiff, setequal, union

```

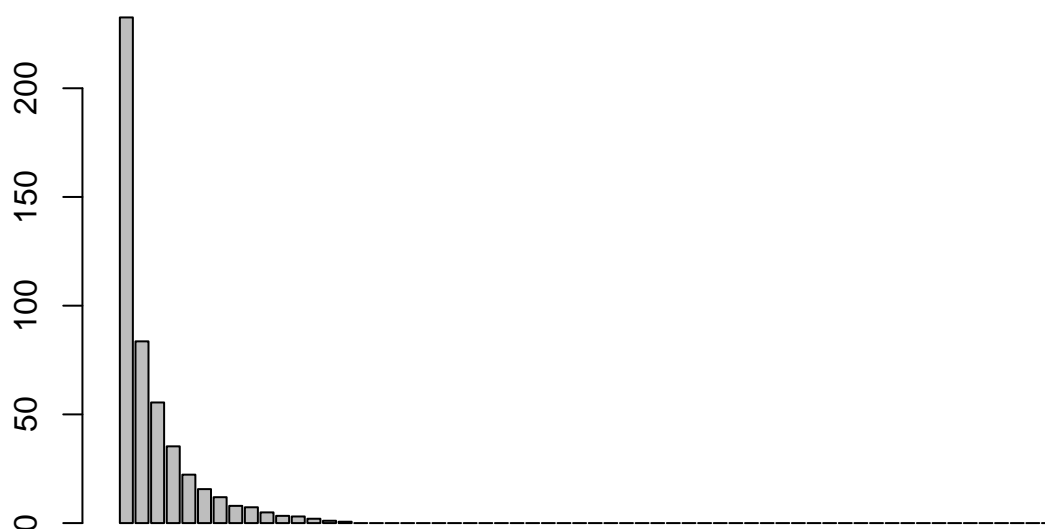
### *The first step: functional PCA*

Eigenvalues of the matrix  $\mathbf{S} = \hat{\Sigma}$

```

## [1] 232.573550 83.587104 55.478879 35.293497 22.268358 15.622057
## [7] 11.912274 7.928606 7.274043 4.912452 3.320067 3.040542
## [13] 2.001118 1.080613 0.624985 0.000000 0.000000 0.000000
## [19] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [25] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [31] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [37] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [43] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [49] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
## [55] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000

```



Proportion of variability:

##	Proportion	Cumulative
## 1	47.76440	47.76440
## 2	17.16656	64.93097
## 3	11.39388	76.32485

Length of the subvectors:

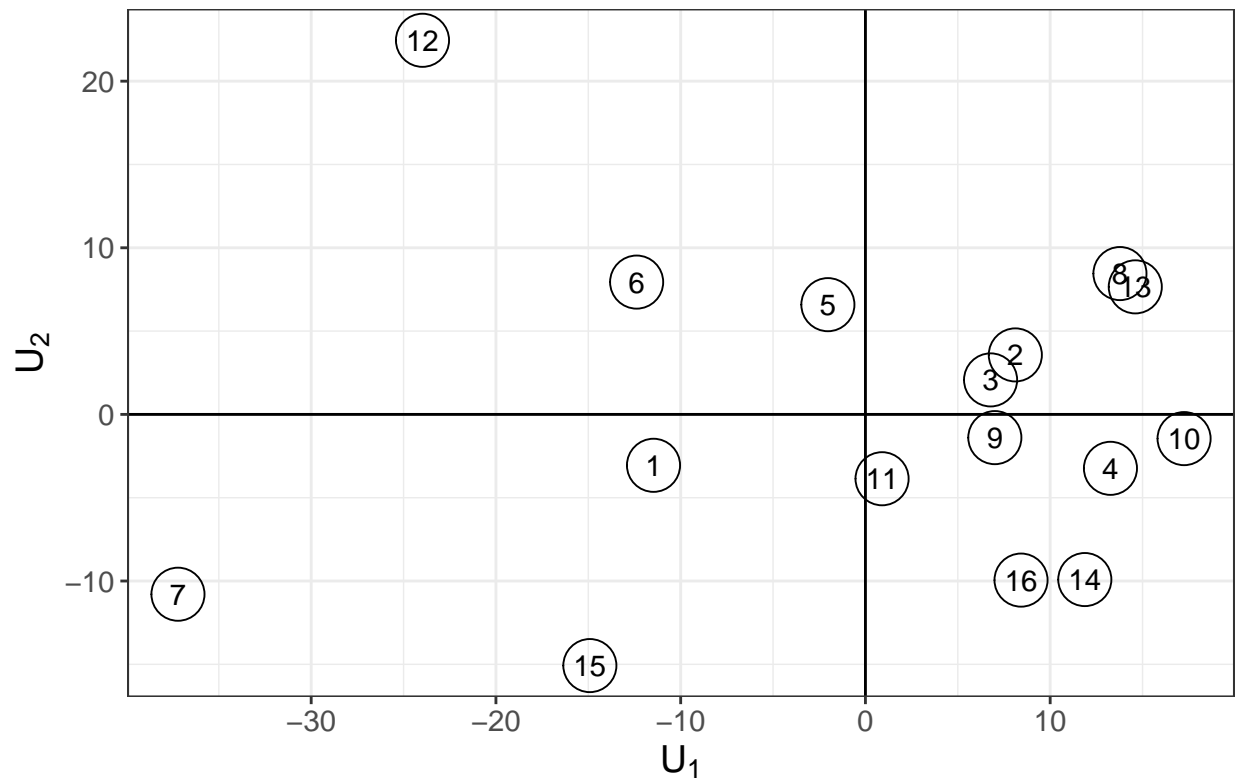
First functional PCA

##	length	%
## 1	0.25250746	6.37600149
## 2	0.30980543	9.59794018
## 3	0.24347875	5.92819019
## 4	0.40492225	16.39620252
## 5	0.31862570	10.15223358
## 6	0.02267171	0.05140063
## 7	0.36228799	13.12525896
## 8	0.36205345	13.10827001
## 9	0.20065735	4.02633718
## 10	0.37906932	14.36935490
## 11	0.21937689	4.81262208
## 12	0.14339415	2.05618828

Second functional PCA

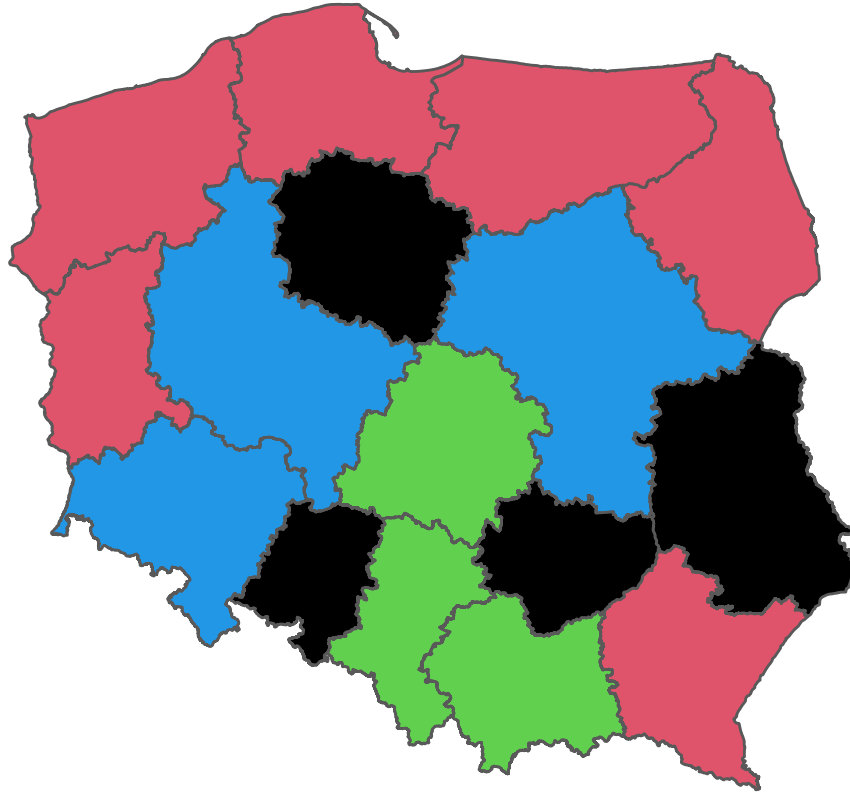
##		length	%
## 1	0.43042482	18.5265524	
## 2	0.13764047	1.8944900	
## 3	0.17619269	3.1043865	
## 4	0.04576930	0.2094829	
## 5	0.32482789	10.5513159	
## 6	0.58719758	34.4801000	
## 7	0.09041789	0.8175394	
## 8	0.08672609	0.7521415	
## 9	0.34280600	11.7515952	
## 10	0.22912073	5.2496308	
## 11	0.27835895	7.7483707	
## 12	0.22168434	4.9143948	

Projections of the 16 regions in a space of the first two functional principal components



Choropleth map:



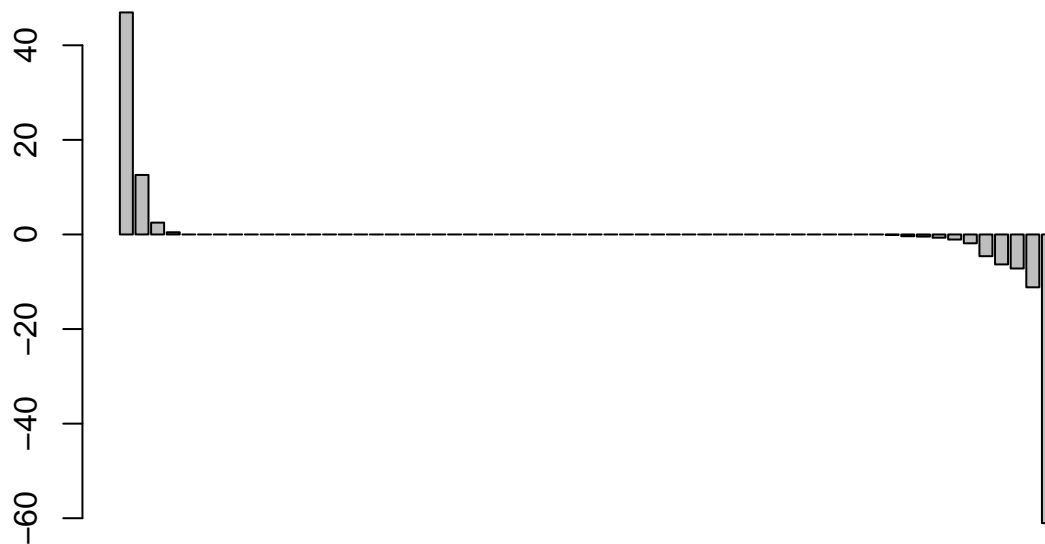


*The second step: spatio-temporal PCA*

Spatial weight matrix No 1;  $k$ -Nearest Neighbour Weights (standard form);  $k=4$

Eigenvalues of the matrix  $C$

##	[1]	46.944242	12.584321	2.502972	0.463481	0.000000	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	-0.101014	-0.370896	-0.449390	-0.683956	-1.073171
##	[55]	-1.872181	-4.594310	-6.334168	-7.196263	-11.173100	-61.040433



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	75.116777	75.11678
## 2	20.136520	95.25330
## 3	4.005075	99.25837

Length of the subvectors:

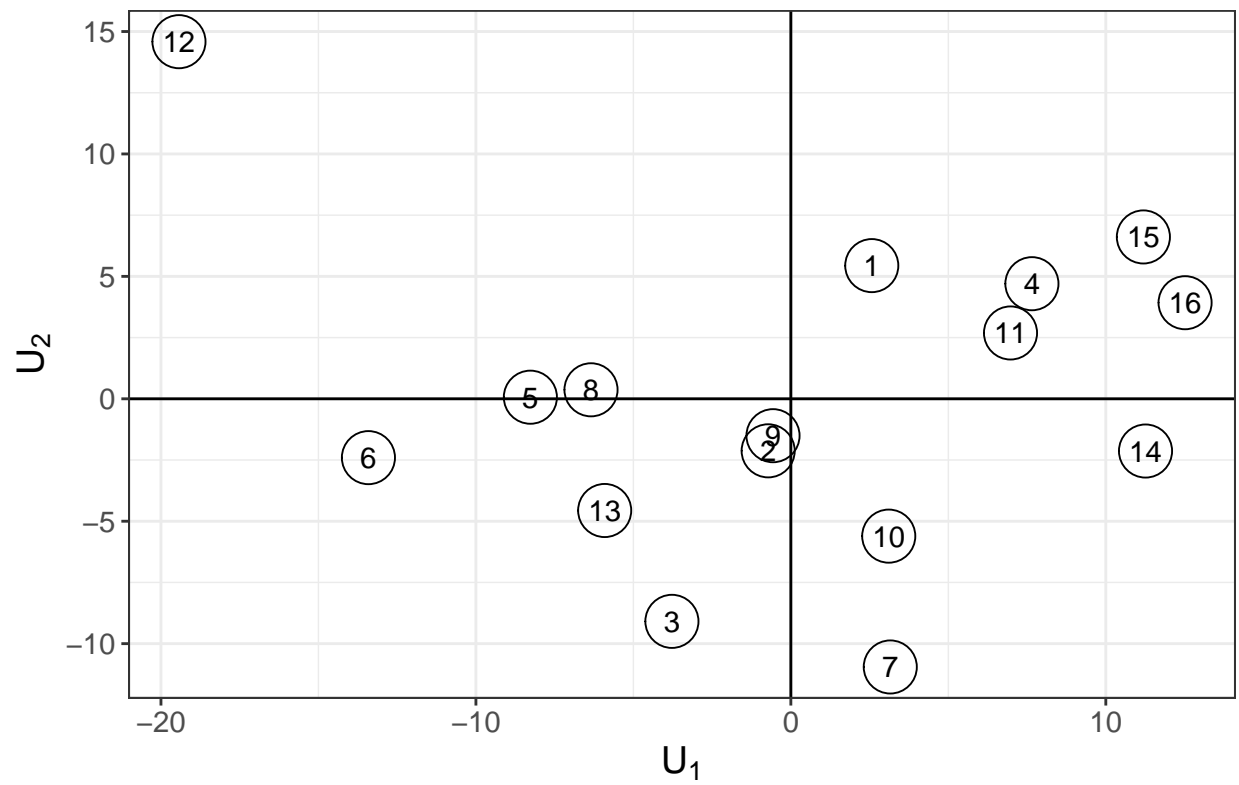
First STPCA

##	length	%
## 1	0.38774054	15.03427254
## 2	0.04547887	0.20683274
## 3	0.26980579	7.27951669
## 4	0.13475614	1.81592160
## 5	0.22164305	4.91256407
## 6	0.73767684	54.41671239
## 7	0.03534287	0.12491184
## 8	0.04020114	0.16161319
## 9	0.28921411	8.36447990
## 10	0.20665796	4.27075129
## 11	0.18314062	3.35404873
## 12	0.02416092	0.05837502

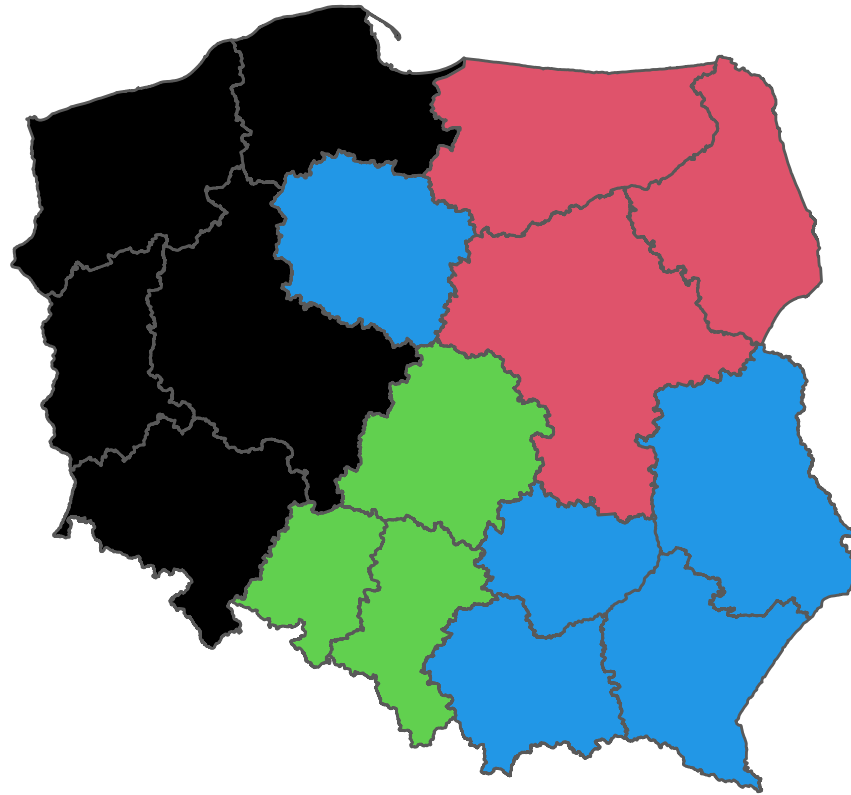
Second STPCA

##	length	%
## 1	0.1547840	2.395808
## 2	0.0680779	0.463460
## 3	0.2826244	7.987656
## 4	0.4468222	19.965008
## 5	0.1618096	2.618233
## 6	0.2689002	7.230734
## 7	0.3324756	11.054005
## 8	0.3459079	11.965226
## 9	0.1262921	1.594969
## 10	0.3258410	10.617233
## 11	0.3718884	13.830099
## 12	0.3205865	10.277568

Projections of the 16 regions in a space of the first two positive spatio-temporal princpal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	64.328329	64.32833
## 59	11.774931	76.10326
## 58	7.583884	83.68714

Length of the subvectors:

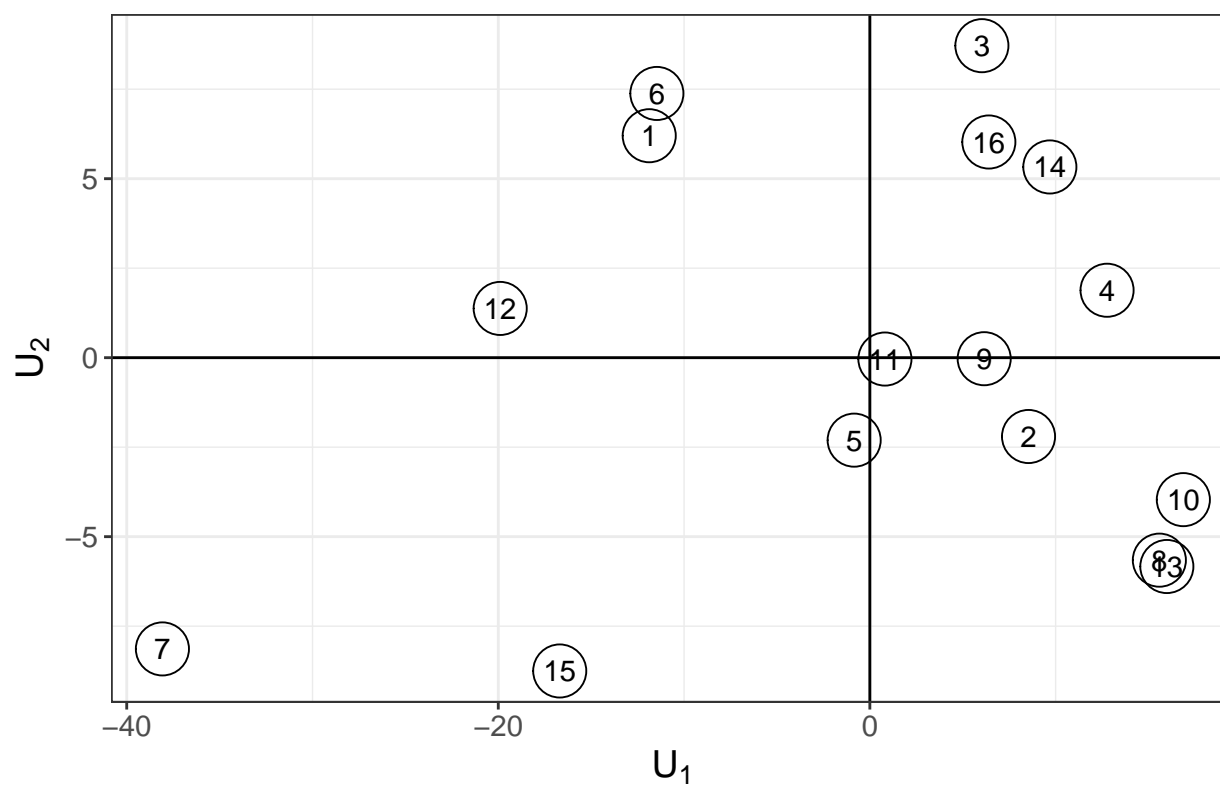
First STPCA

##	length	%
## 1	0.19050032	3.6290372
## 2	0.32867807	10.8029275
## 3	0.23476976	5.5116841
## 4	0.33728440	11.3760767
## 5	0.34920984	12.1947512
## 6	0.09520081	0.9063194
## 7	0.38781911	15.0403661
## 8	0.38910460	15.1402388
## 9	0.19068217	3.6359691
## 10	0.33742905	11.3858361
## 11	0.26985657	7.2822569
## 12	0.17591296	3.0945369

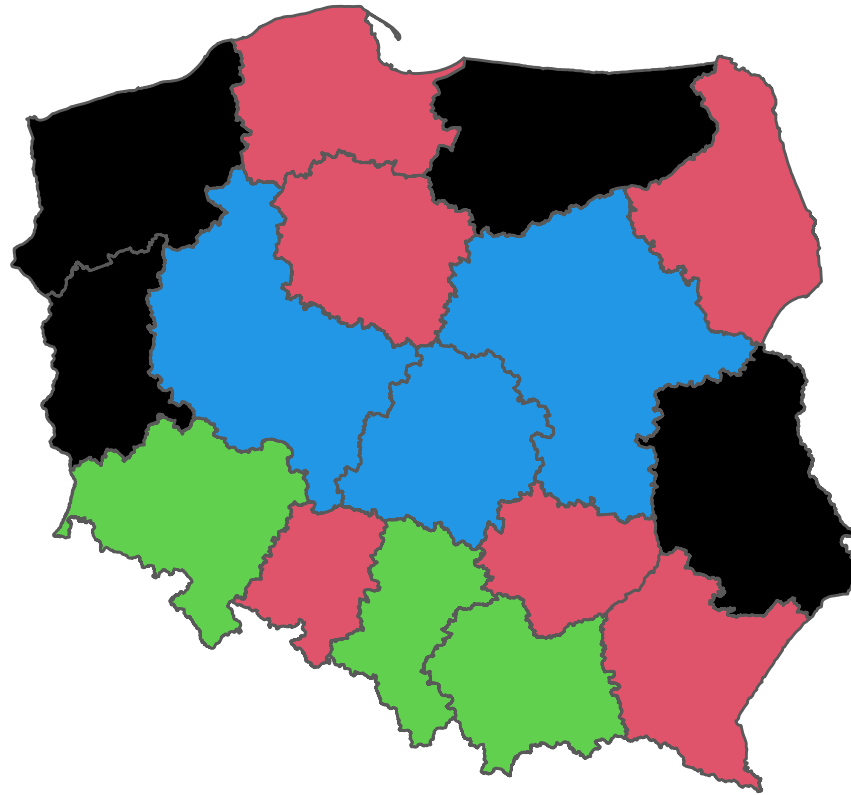
## Second STPCA

##	length	%
## 1	0.05744647	0.3300096
## 2	0.31913245	10.1845523
## 3	0.55948560	31.3024139
## 4	0.33930267	11.5126302
## 5	0.35584470	12.6625453
## 6	0.36702495	13.4707310
## 7	0.09292357	0.8634791
## 8	0.09980990	0.9962016
## 9	0.17660601	3.1189682
## 10	0.04797400	0.2301505
## 11	0.20217623	4.0875228
## 12	0.33527296	11.2407955

Projections of the 16 regions in a space of the first two negative spatio-temporal principal components



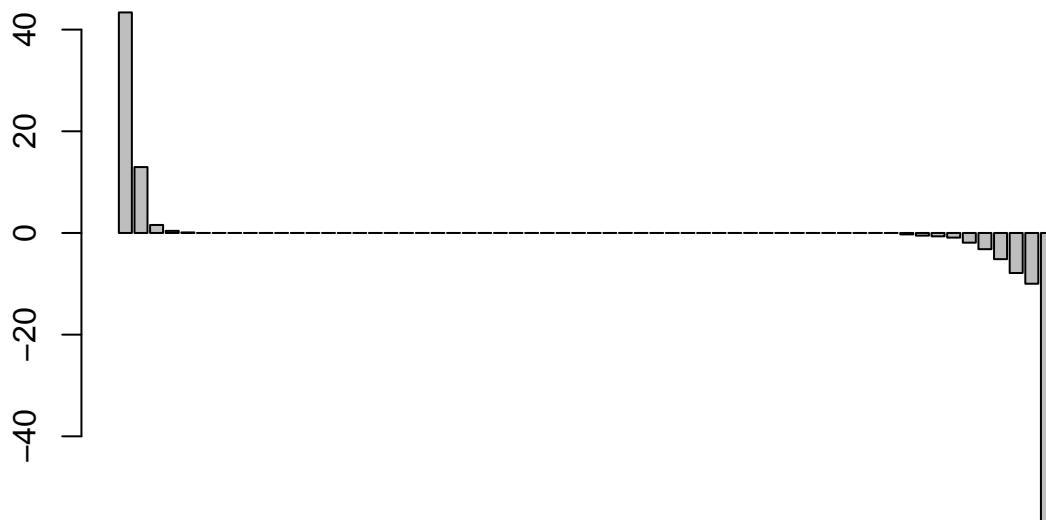
Choropleth map



Spatial weight matrix No 2;  $k$ -Nearest Neighbour Weights (symmetric form);  $k=4$

Eigenvalues of the matrix  $\mathbf{C}$

##	[1]	43.380217	12.961590	1.575696	0.413603	0.100953	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	0.000000	-0.275640	-0.522909	-0.659916	-0.911000
##	[55]	-1.907947	-3.187309	-5.148355	-7.873261	-9.985458	-57.077666



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	74.24044	74.24044
## 2	22.18233	96.42277
## 3	2.69663	99.11940

Length of the subvectors:

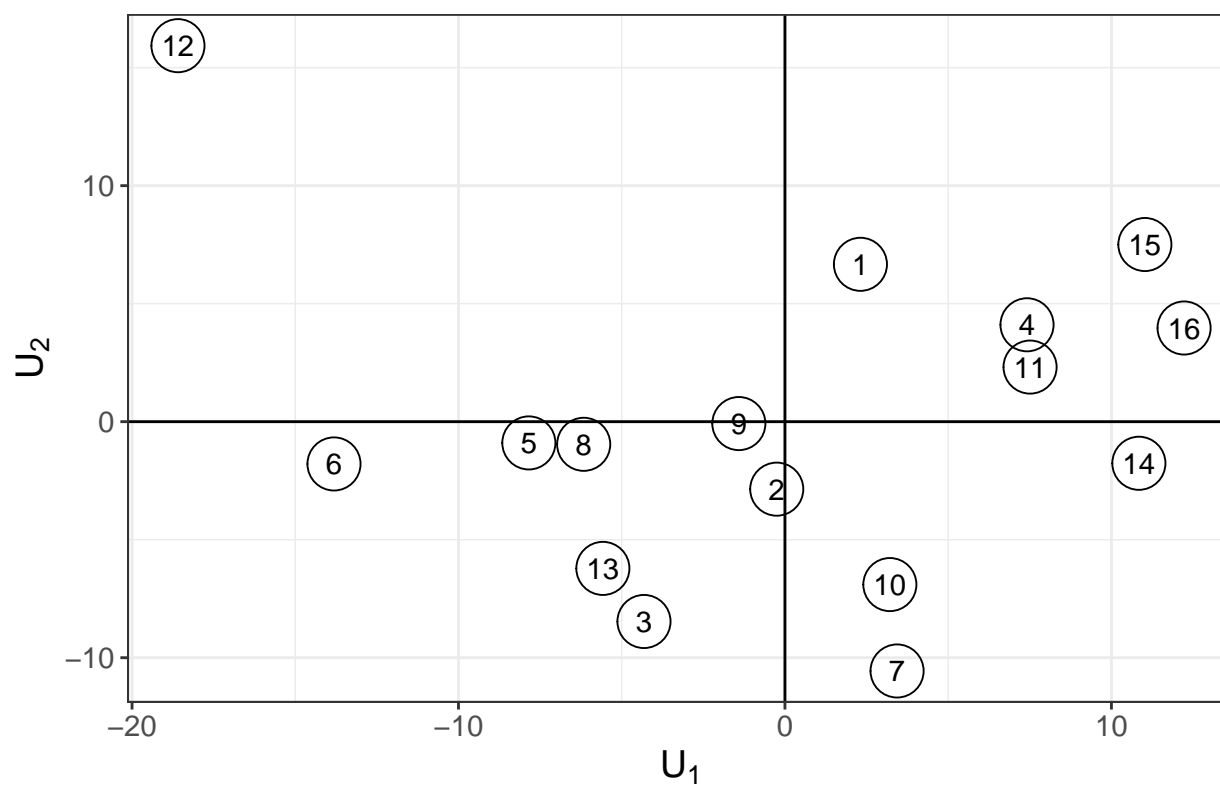
First STPCA

##	length	%
## 1	0.37119051	13.7782392
## 2	0.04608011	0.2123376
## 3	0.30793378	9.4823213
## 4	0.15947530	2.5432372
## 5	0.22703908	5.1546745
## 6	0.73564169	54.1168694
## 7	0.03176274	0.1008872
## 8	0.03782888	0.1431024
## 9	0.27275764	7.4396728
## 10	0.17755162	3.1524576
## 11	0.19313795	3.7302266
## 12	0.03820656	0.1459742

## Second STPCA

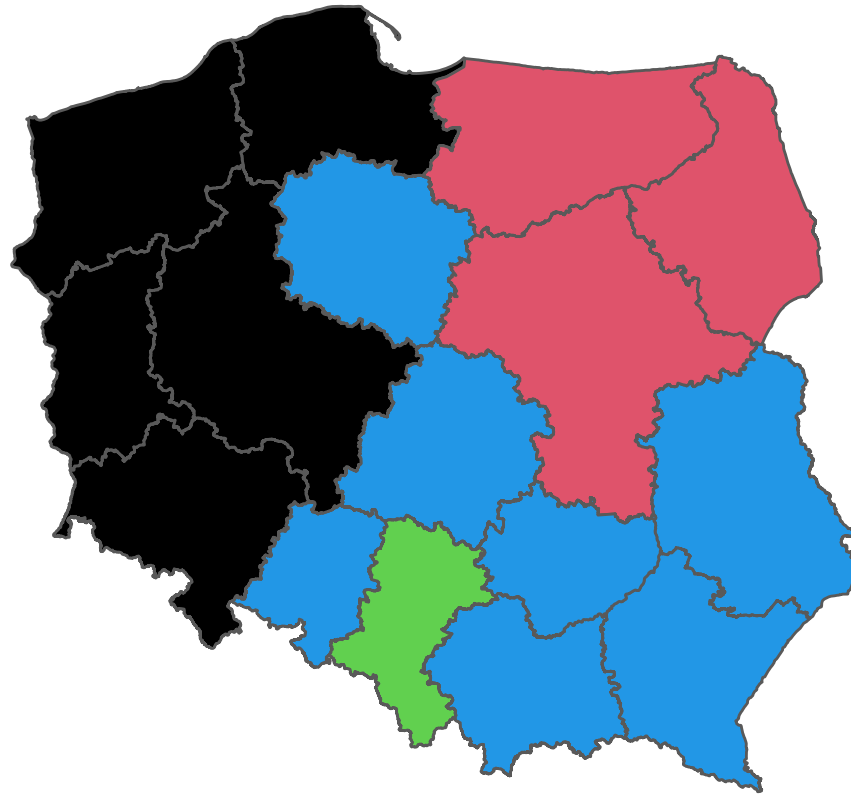
##		length	%
## 1	0.17849899	3.1861888	
## 2	0.09045159	0.8181491	
## 3	0.33639509	11.3161659	
## 4	0.42300701	17.8934932	
## 5	0.20632479	4.2569920	
## 6	0.30776423	9.4718822	
## 7	0.27398960	7.5070299	
## 8	0.28664192	8.2163593	
## 9	0.12505197	1.5637995	
## 10	0.30824988	9.5017991	
## 11	0.33725442	11.3740541	
## 12	0.38592858	14.8940870	

Projections of the 16 regions in a space of the first two positive spatio-temporal principal components



Choropleth map





Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	65.194766	65.19477
## 59	11.405505	76.60027
## 58	8.992929	85.59320

Length of the subvectors:

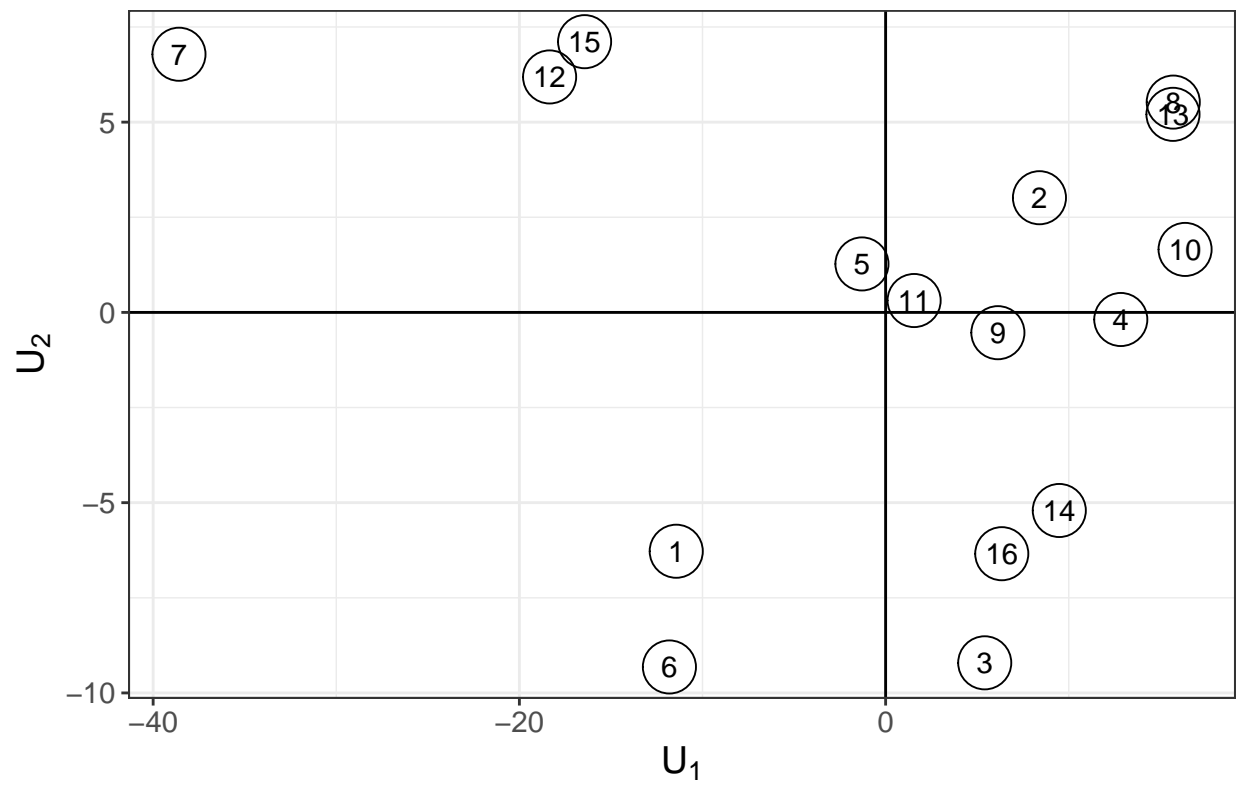
First STPCA

##	length	%
## 1	0.16599408	2.7554035
## 2	0.34196004	11.6936667
## 3	0.24195818	5.8543761
## 4	0.29432058	8.6624606
## 5	0.38218733	14.6067156
## 6	0.07028624	0.4940155
## 7	0.38724921	14.9961954
## 8	0.38878942	15.1157215
## 9	0.19367662	3.7510634
## 10	0.33515093	11.2326148
## 11	0.28032548	7.8582377
## 12	0.17261313	2.9795292

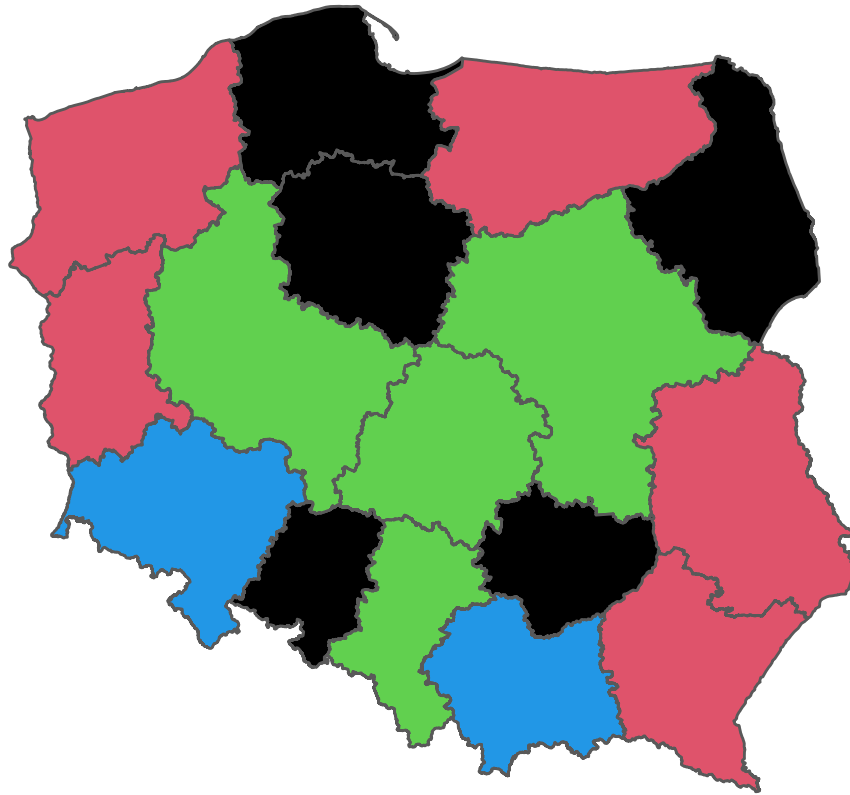
Second STPCA

##	length	%
## 1	0.04002117	0.1601694
## 2	0.51145701	26.1588273
## 3	0.50933531	25.9422455
## 4	0.40826541	16.6680643
## 5	0.26989452	7.2843050
## 6	0.29882627	8.9297138
## 7	0.13083440	1.7117641
## 8	0.12506299	1.5640750
## 9	0.15556954	2.4201881
## 10	0.10114509	1.0230328
## 11	0.14551164	2.1173638
## 12	0.24536199	6.0202508

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



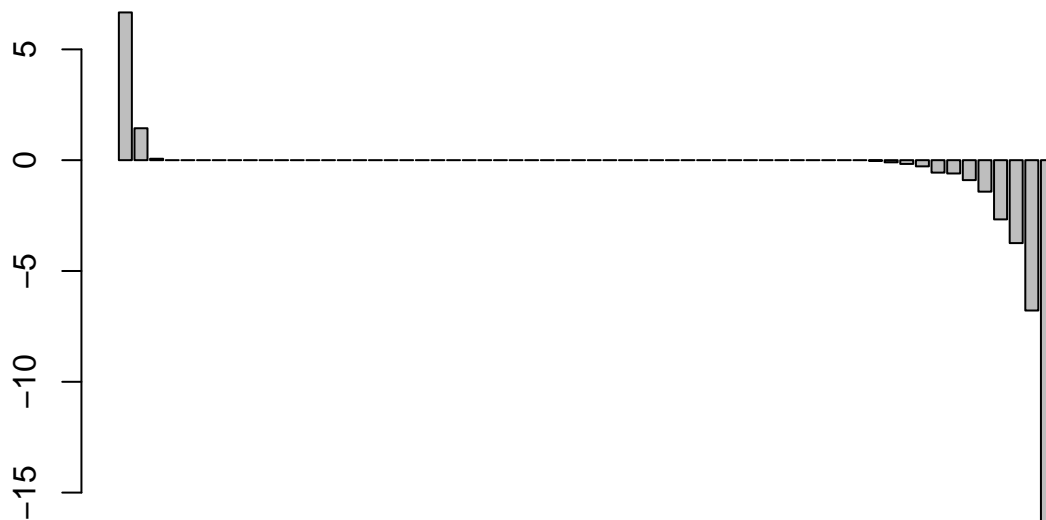
Choropleth map



### Spatial weight matrix No 3; Radial Distance Weights

Eigenvalues of the matrix  $\mathbf{C}$

##	[1]	6.666403	1.440262	0.064082	0.000000	0.000000	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	-0.031964	-0.097912	-0.169292	-0.279051	-0.562150	-0.599872
##	[55]	-0.899573	-1.420646	-2.672895	-3.742258	-6.783801	-16.378353



Positive spatio-temporal principal components

Proportion of variability:

```
##   Proportion Cumulative
## 1 81.5886618   81.58866
## 2 17.6270526   99.21571
## 3  0.7842857  100.00000
```

Length of the subvectors:

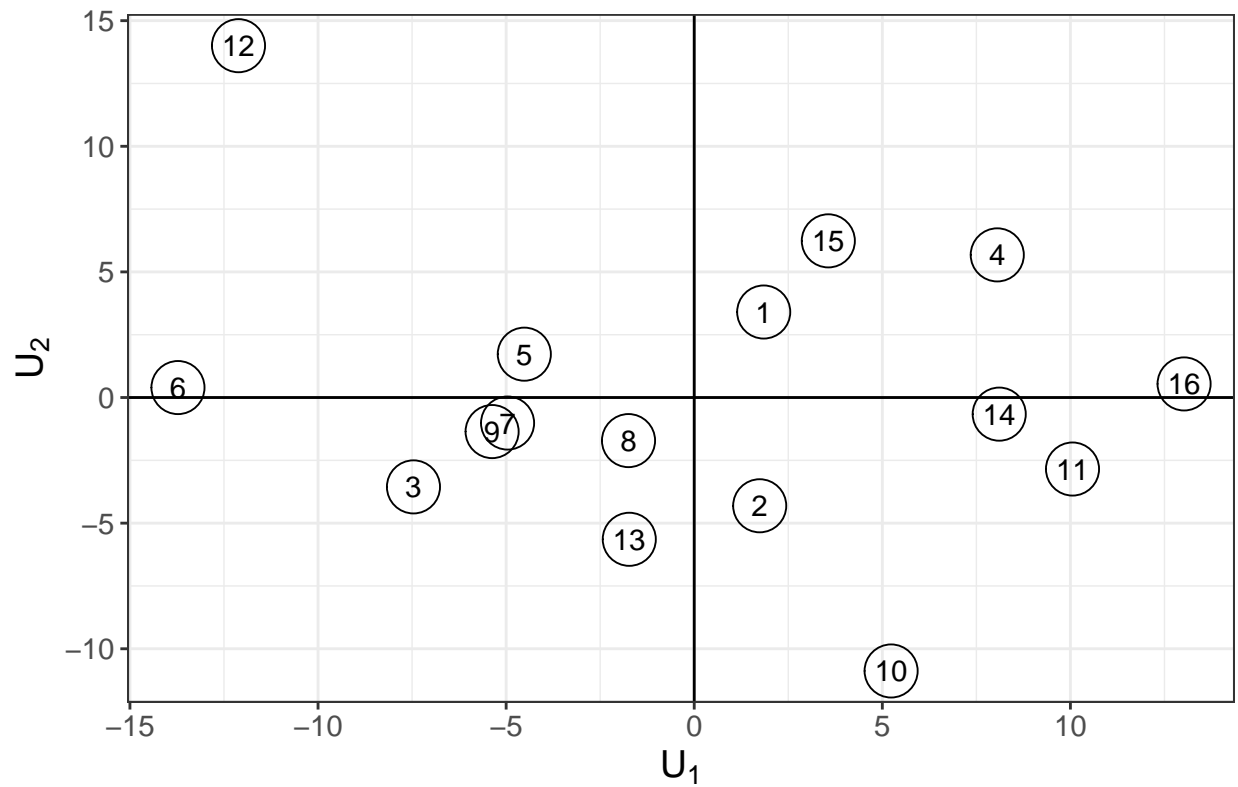
First STPCA

```
##      length      %
## 1 0.23052908  5.31436576
## 2 0.09769375  0.95440684
## 3 0.35135629 12.34512443
## 4 0.23978276  5.74957697
## 5 0.07887382  0.62210788
## 6 0.71501736 51.12498201
## 7 0.19302068  3.72569842
## 8 0.20496862  4.20121360
## 9 0.15690536  2.46192927
## 10 0.03062082  0.09376343
## 11 0.25908025  6.71225784
## 12 0.25873874  6.69457355
```

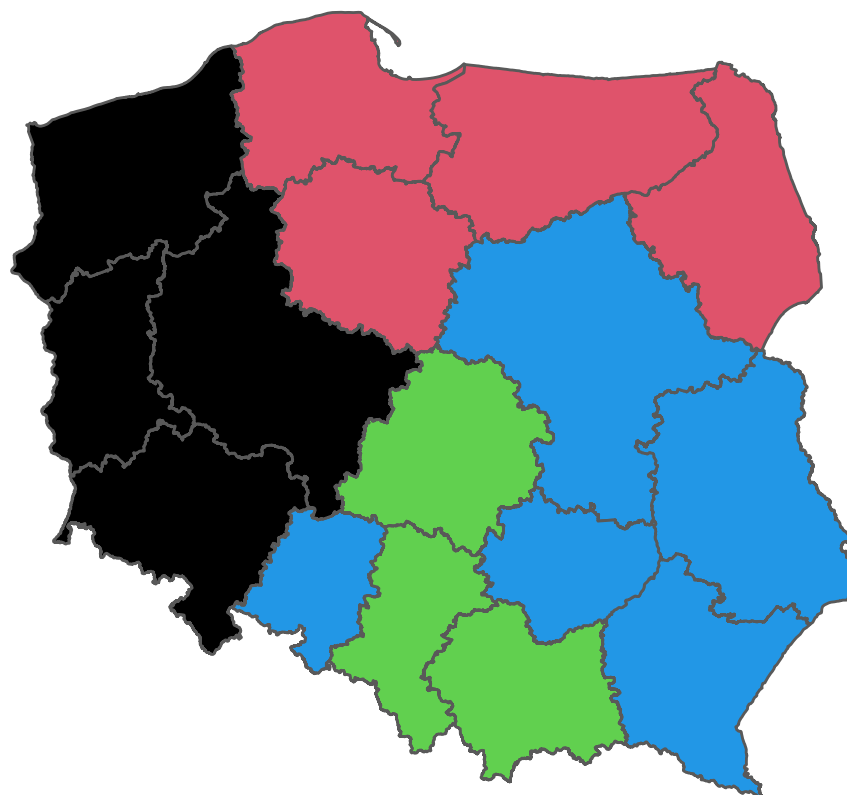
## Second STPCA

##		length	%
## 1	0.02123676	0.04510002	
## 2	0.32184855	10.35864903	
## 3	0.57998246	33.63796580	
## 4	0.34656306	12.01059567	
## 5	0.06928514	0.48004308	
## 6	0.11498436	1.32214024	
## 7	0.22051218	4.86256236	
## 8	0.23433272	5.49118232	
## 9	0.21957868	4.82147959	
## 10	0.42096083	17.72080212	
## 11	0.21003691	4.41155030	
## 12	0.21995294	4.83792947	

Projections of the 16 regions in a space of the first two positive spatio-temporal principal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	48.69037	48.69037
## 59	20.16722	68.85759
## 58	11.12517	79.98275

Length of the subvectors:

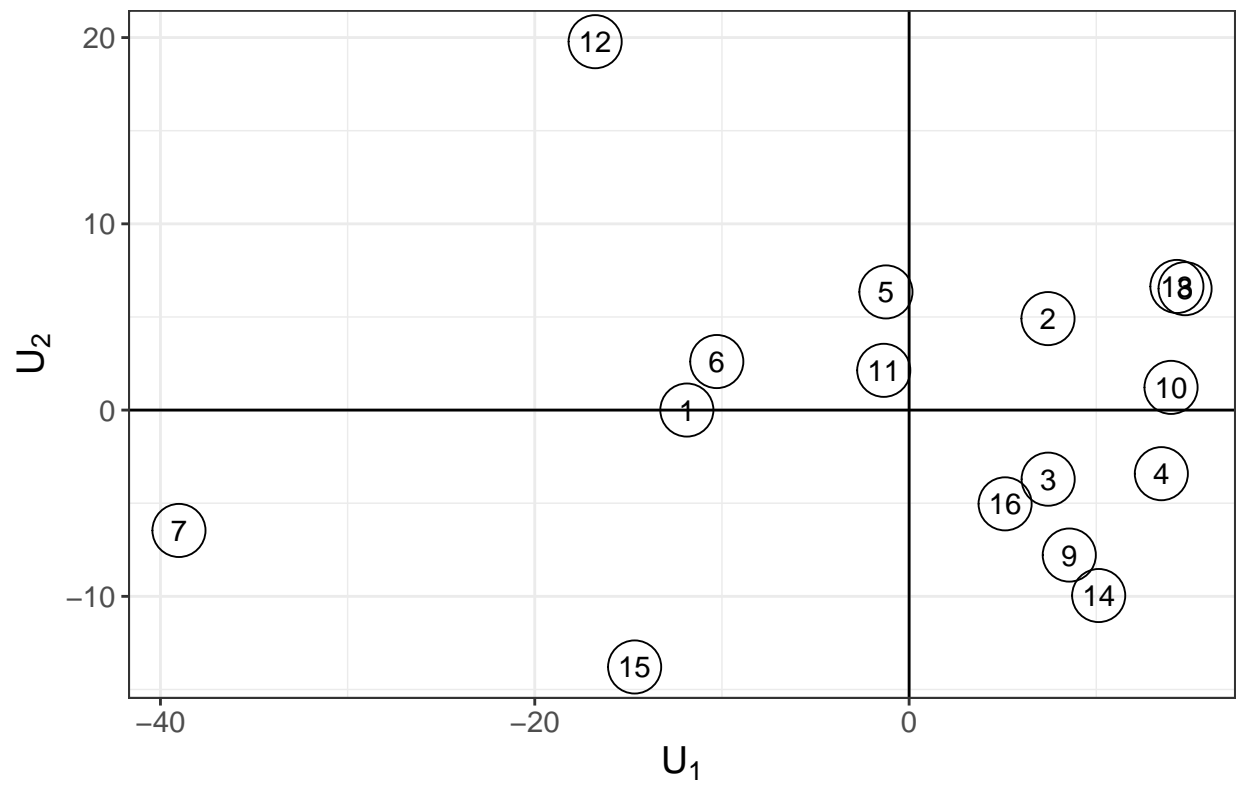
First STPCA

##	length	%
## 1	0.1581017	2.499615
## 2	0.4649847	21.621073
## 3	0.1118803	1.251720
## 4	0.3520151	12.391460
## 5	0.3777498	14.269491
## 6	0.1256914	1.579834
## 7	0.3733027	13.935493
## 8	0.3710206	13.765627
## 9	0.2138813	4.574519
## 10	0.3224052	10.394514
## 11	0.1603657	2.571717
## 12	0.1070017	1.144936

Second STPCA

##	length	%
## 1	0.3506999	12.299043
## 2	0.1128478	1.273463
## 3	0.1400892	1.962499
## 4	0.1974029	3.896789
## 5	0.2461521	6.059084
## 6	0.3060214	9.364909
## 7	0.1110842	1.233971
## 8	0.1084992	1.177207
## 9	0.2856770	8.161134
## 10	0.3368171	11.344574
## 11	0.4904342	24.052566
## 12	0.4378900	19.174762

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



Choropleth map



Spatial weight matrix No 4; Power Distance Weights;  $\alpha = 2$

Eigenvalues of the matrix  $\mathbf{C}$

##	[1]	49.792228	4.563935	1.006782	0.493476	0.037245	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	0.000000	-0.042284	-0.274903	-0.340255	-0.503706
##	[55]	-1.126102	-2.652476	-3.253162	-5.686046	-18.233271	-48.496268





Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	89.083847	89.08385
## 2	8.165389	97.24924
## 3	1.801246	99.05048

Length of the subvectors:

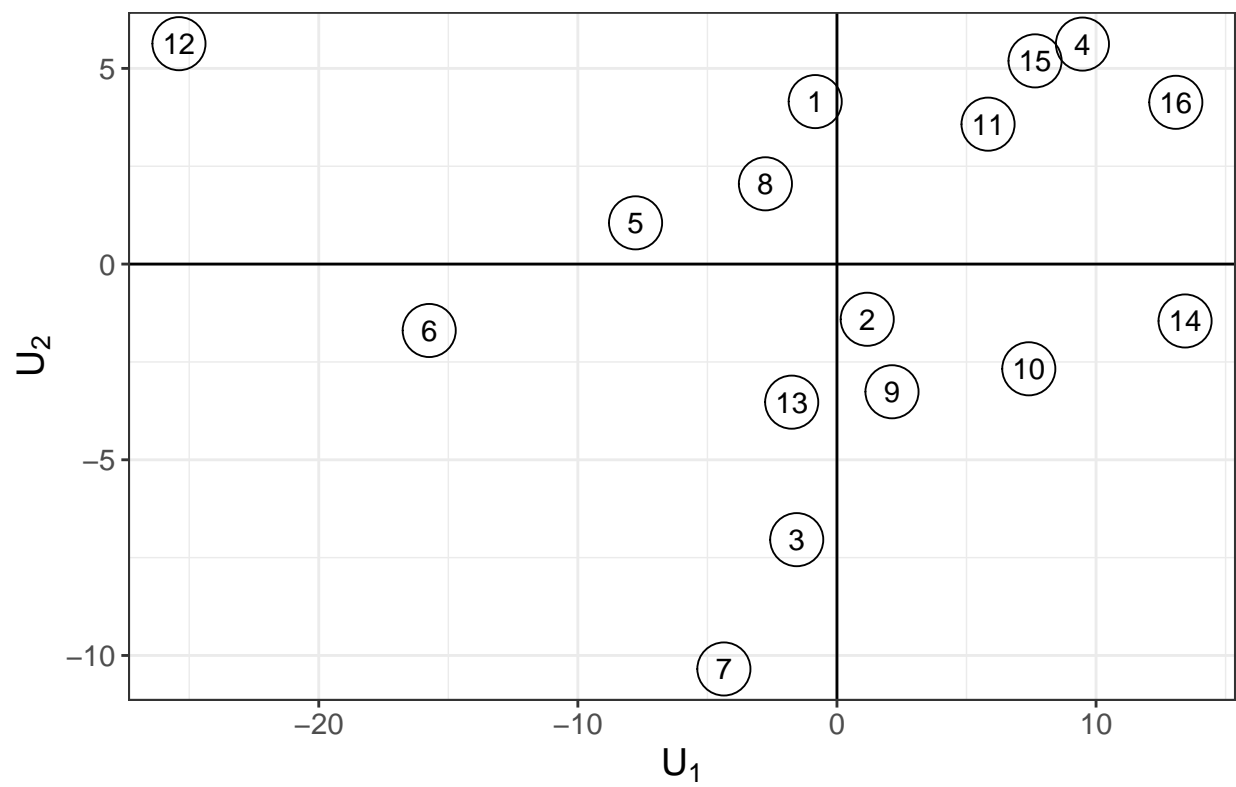
First STPCA

##	length	%
## 1	0.45732757	20.9148511
## 2	0.10992807	1.2084180
## 3	0.34214793	11.7065208
## 4	0.04199905	0.1763920
## 5	0.13581329	1.8445248
## 6	0.64090192	41.0755275
## 7	0.05932957	0.3519998
## 8	0.06504582	0.4230958
## 9	0.33869021	11.4711055
## 10	0.28756298	8.2692466
## 11	0.15605968	2.4354625
## 12	0.03505076	0.1228556

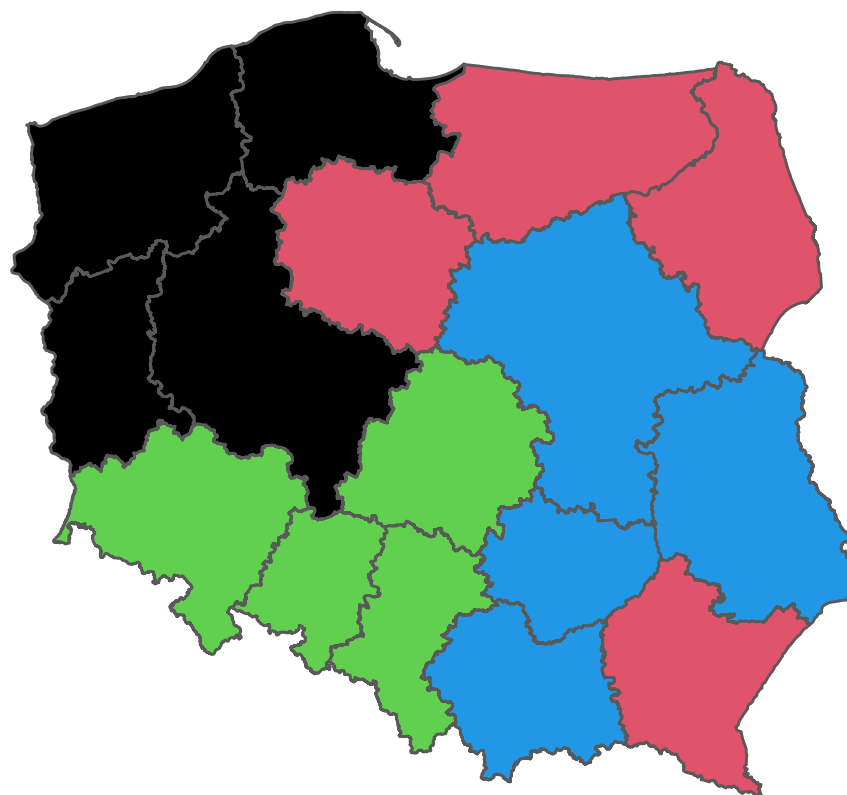
Second STPCA

##	length	%
## 1	0.005127776	0.002629409
## 2	0.149059968	2.221887420
## 3	0.269415878	7.258491514
## 4	0.458520905	21.024142026
## 5	0.108331678	1.173575251
## 6	0.198691795	3.947842924
## 7	0.437454199	19.136617606
## 8	0.451180463	20.356380975
## 9	0.109660770	1.202548449
## 10	0.232797541	5.419469522
## 11	0.367917725	13.536345255
## 12	0.217257213	4.720069649

Projections of the 16 regions in a space of the first two positive spatio-temporal princpal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	60.162743	60.16274
## 59	22.619547	82.78229
## 58	7.053906	89.83620

Length of the subvectors:

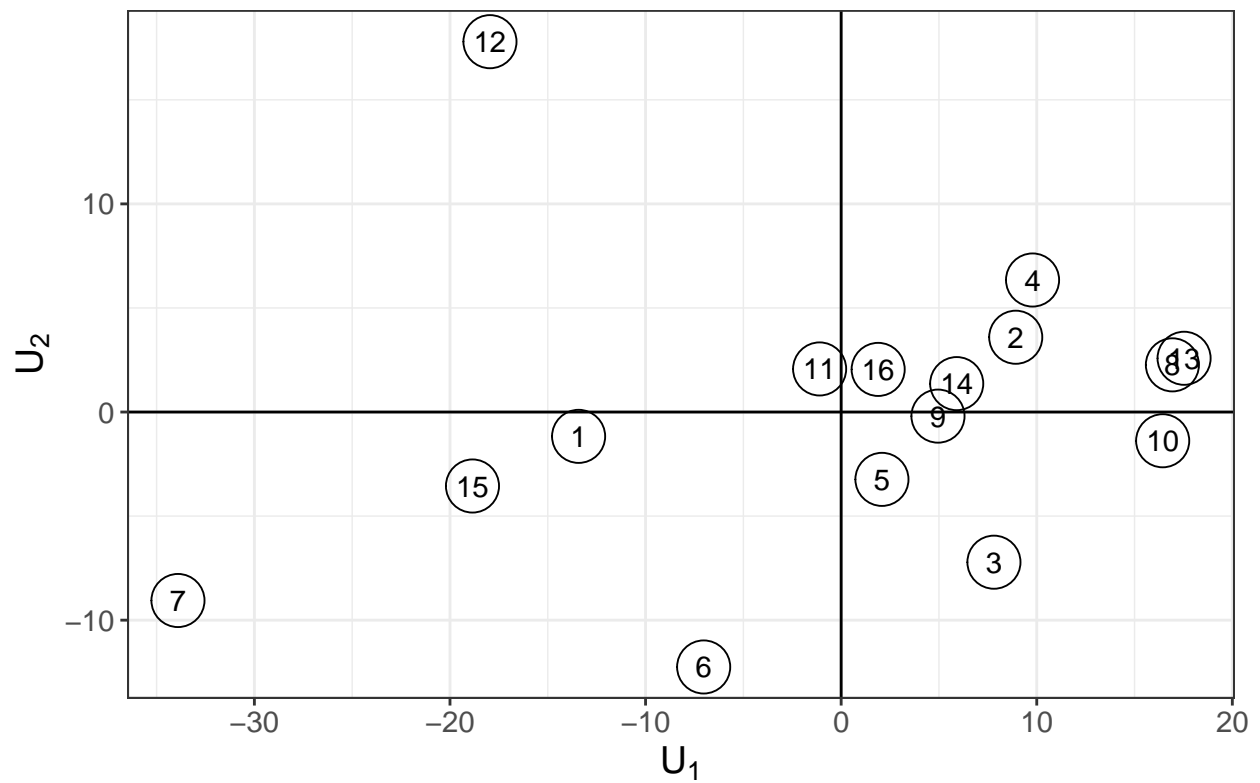
First STPCA

##	length	%
## 1	0.1500738	2.252213
## 2	0.2742939	7.523714
## 3	0.1955719	3.824836
## 4	0.3677090	13.520987
## 5	0.2774037	7.695282
## 6	0.3590030	12.888312
## 7	0.3741676	14.000143
## 8	0.3769579	14.209723
## 9	0.1918697	3.681397
## 10	0.3054514	9.330054
## 11	0.2423613	5.873899
## 12	0.2280228	5.199438

## Second STPCA

##		length	%
## 1	0.2330116	5.429440	
## 2	0.6435056	41.409945	
## 3	0.2763171	7.635116	
## 4	0.2635271	6.944651	
## 5	0.2652935	7.038067	
## 6	0.2350199	5.523433	
## 7	0.1760147	3.098119	
## 8	0.1709823	2.923494	
## 9	0.2468407	6.093032	
## 10	0.1953836	3.817477	
## 11	0.2352779	5.535568	
## 12	0.2133461	4.551657	

Projections of the 16 regions in a space of the first two negative spatio-temporal principal components



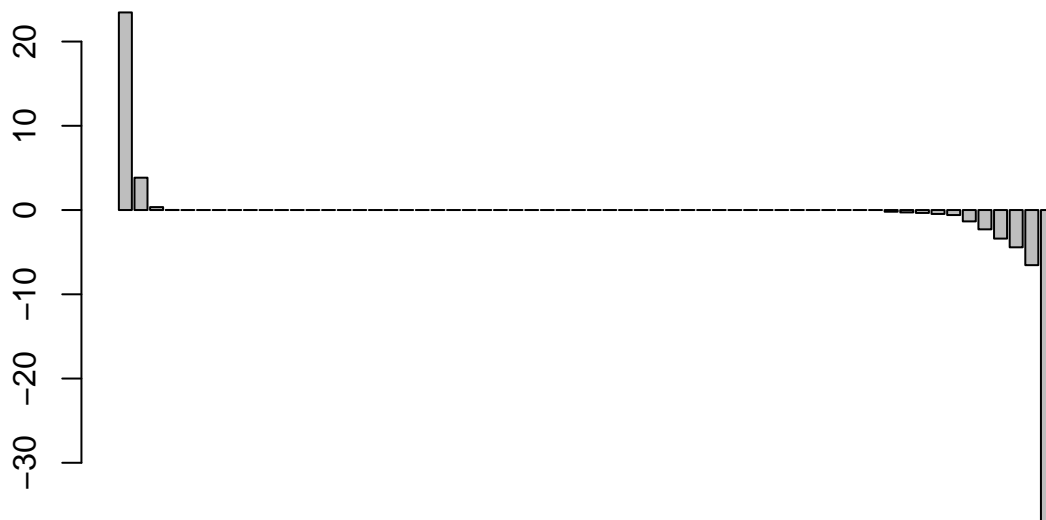
Choropleth map



Spatial weight matrix No 5; Exponential Distance Weights;  $\alpha = 2$

Eigenvalues of the matrix  $\mathbf{C}$

##	[1]	23.457955	3.839739	0.346754	0.008763	0.000000	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	-0.198457	-0.282684	-0.345196	-0.465788	-0.595617
##	[55]	-1.347061	-2.290615	-3.391679	-4.425314	-6.542686	-37.167880



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	84.829048	84.82905
## 2	13.885327	98.71437
## 3	1.253937	99.96831

Length of the subvectors:

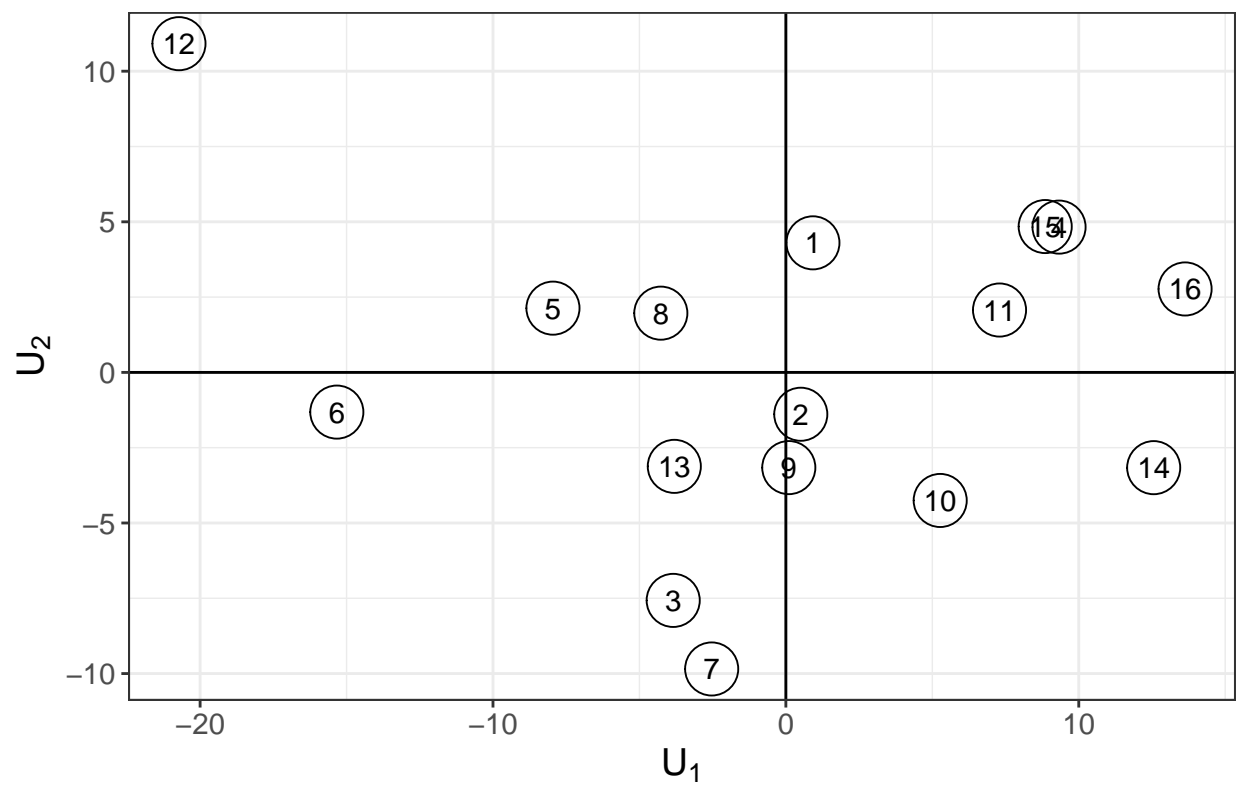
First STPCA

##	length	%
## 1	0.38325693	14.6885877
## 2	0.07403810	0.5481641
## 3	0.31326649	9.8135896
## 4	0.10086553	1.0173855
## 5	0.15304443	2.3422597
## 6	0.74527796	55.5439233
## 7	0.05842161	0.3413085
## 8	0.06496247	0.4220123
## 9	0.28867709	8.3334463
## 10	0.21242443	4.5124136
## 11	0.15254170	2.3268970
## 12	0.03316811	0.1100124

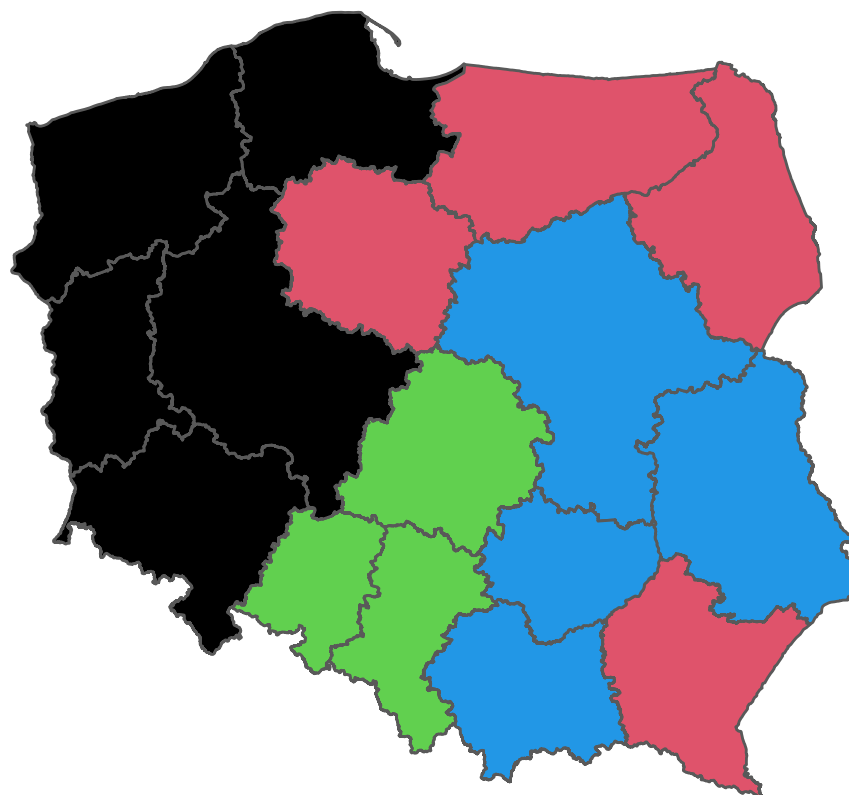
Second STPCA

##	length	%
## 1	0.05521171	0.3048332
## 2	0.07257833	0.5267614
## 3	0.30487798	9.2950584
## 4	0.46120191	21.2707201
## 5	0.11870849	1.4091706
## 6	0.12987050	1.6866346
## 7	0.40064428	16.0515840
## 8	0.41513593	17.2337838
## 9	0.11263611	1.2686893
## 10	0.34116607	11.6394287
## 11	0.36883046	13.6035911
## 12	0.23895072	5.7097446

Projections of the 16 regions in a space of the first two positive spatio-temporal princpal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	65.146257	65.14626
## 59	11.467737	76.61399
## 58	7.756499	84.37049

Length of the subvectors:

First STPCA

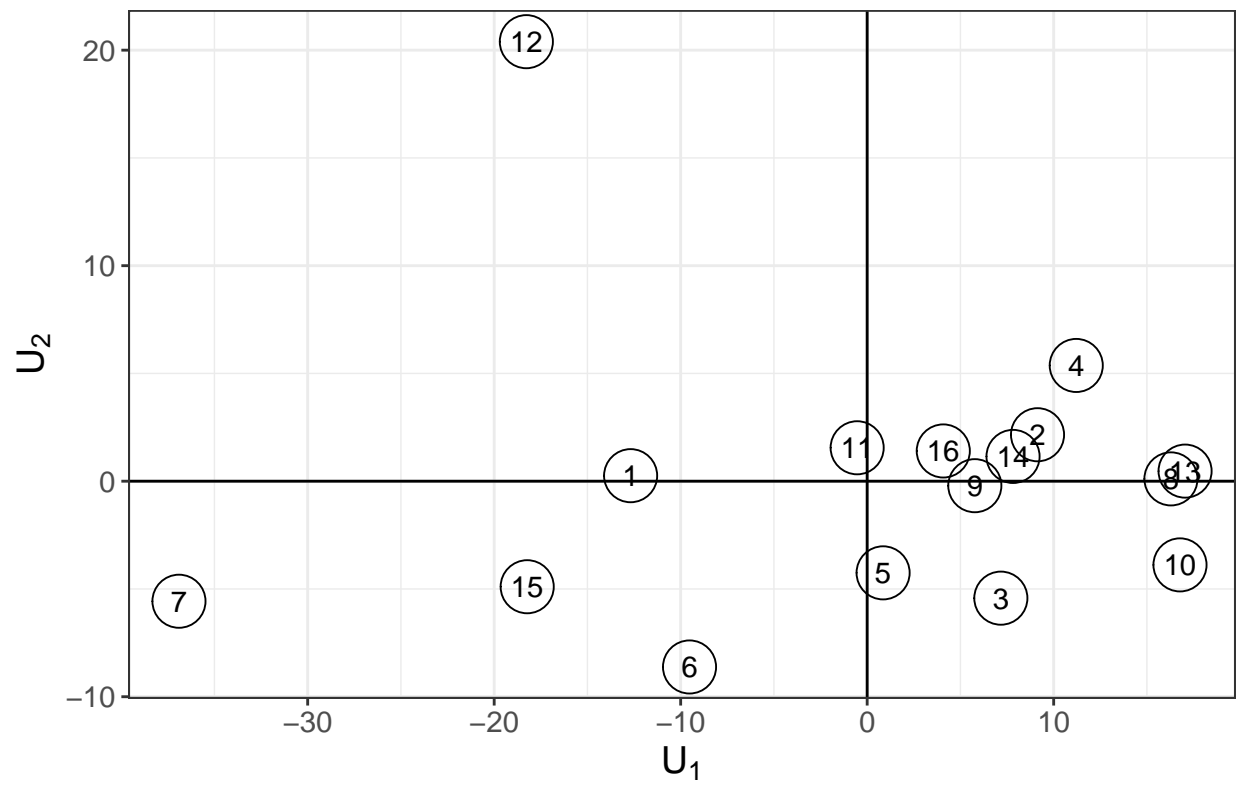
##	length	%
## 1	0.1617586	2.616584
## 2	0.3209275	10.299443
## 3	0.2201086	4.844779
## 4	0.3674992	13.505569
## 5	0.3423138	11.717877
## 6	0.2393403	5.728377
## 7	0.3741530	13.999045
## 8	0.3748456	14.050922
## 9	0.1861416	3.464869
## 10	0.3013000	9.078171
## 11	0.2569981	6.604801
## 12	0.2022267	4.089563



Second STPCA

##	length	%
## 1	0.2965897	8.796547
## 2	0.5936252	35.239088
## 3	0.1837281	3.375603
## 4	0.2045480	4.183988
## 5	0.2802818	7.855786
## 6	0.2665649	7.105685
## 7	0.2563553	6.571804
## 8	0.2559801	6.552581
## 9	0.2863694	8.200743
## 10	0.1524953	2.325481
## 11	0.1939171	3.760385
## 12	0.2456076	6.032309

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



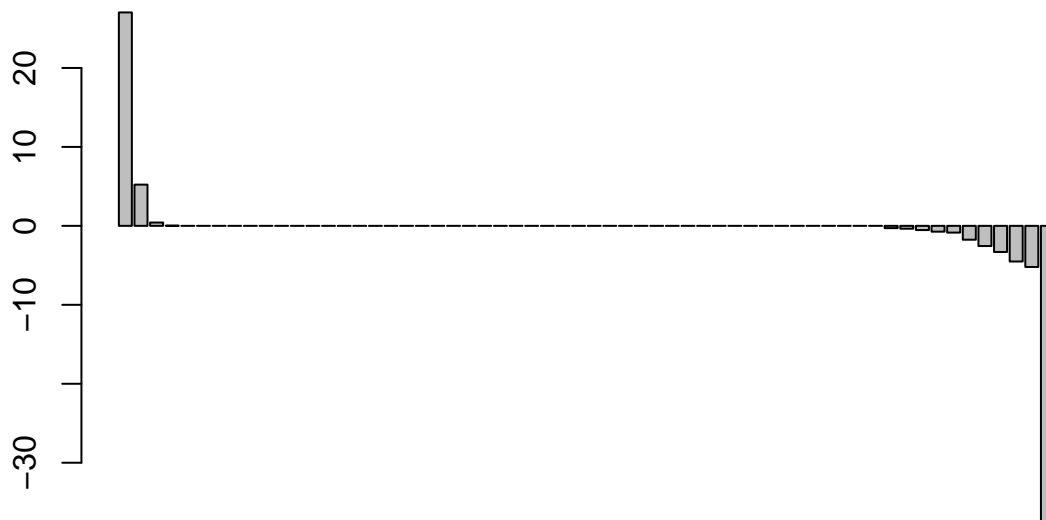
Choropleth map



Spatial weight matrix No 6; Double-Power Distance Weights; k=3

Eigenvalues of the matrix  $\mathbf{C}$

##	[1]	27.030521	5.221910	0.419323	0.024862	0.000000	0.000000
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	-0.289521	-0.360686	-0.533322	-0.736776	-0.855823
##	[55]	-1.754552	-2.555917	-3.311224	-4.515276	-5.207588	-37.652988



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	82.670700	82.67070
## 2	15.970796	98.64150
## 3	1.282466	99.92396

Length of the subvectors:

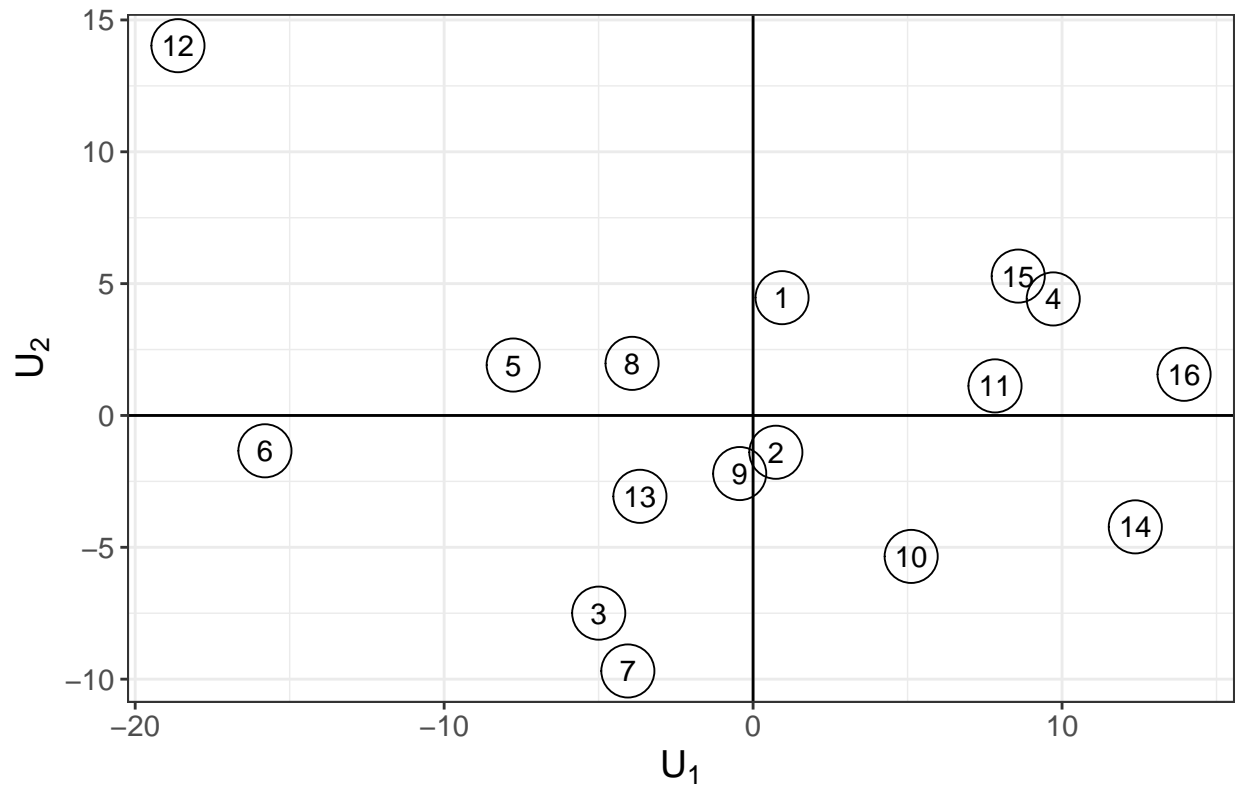
First STPCA

##	length	%
## 1	0.33928281	11.5112827
## 2	0.08768963	0.7689470
## 3	0.32425959	10.5144279
## 4	0.13116957	1.7205457
## 5	0.13512568	1.8258950
## 6	0.77366736	59.8561177
## 7	0.08352494	0.6976415
## 8	0.08989297	0.8080745
## 9	0.25757880	6.6346839
## 10	0.17523833	3.0708474
## 11	0.15321750	2.3475603
## 12	0.04939396	0.2439763

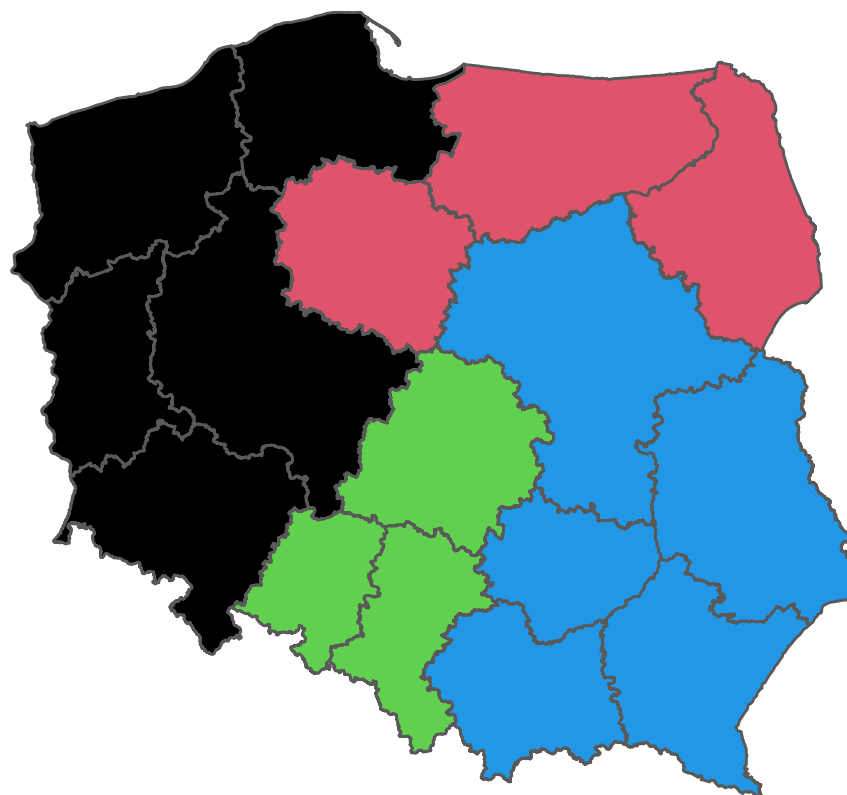
## Second STPCA

##	length	%
## 1	0.09081816	0.8247938
## 2	0.06983898	0.4877484
## 3	0.33052434	10.9246337
## 4	0.47018704	22.1075852
## 5	0.14039441	1.9710592
## 6	0.08862736	0.7854808
## 7	0.35079635	12.3058079
## 8	0.36518247	13.3358233
## 9	0.14101807	1.9886096
## 10	0.36571519	13.3747600
## 11	0.35730682	12.7668161
## 12	0.30210730	9.1268821

Projections of the 16 regions in a space of the first two positive spatio-temporal principal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	65.173263	65.17326
## 59	9.013774	74.18704
## 58	7.815457	82.00249

Length of the subvectors:

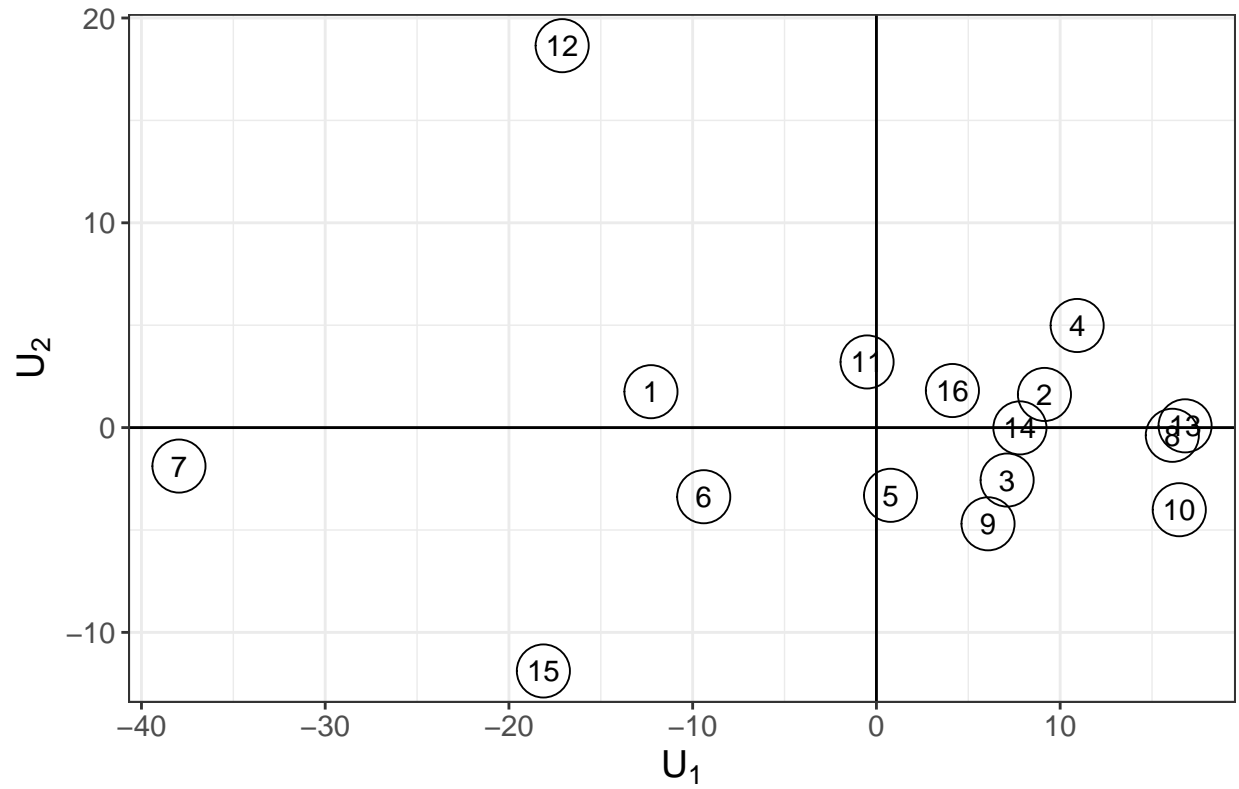
First STPCA

##	length	%
## 1	0.1342567	1.802486
## 2	0.3198942	10.233228
## 3	0.2192357	4.806430
## 4	0.3713936	13.793320
## 5	0.3806777	14.491551
## 6	0.2252119	5.072038
## 7	0.3761142	14.146189
## 8	0.3754274	14.094571
## 9	0.1711254	2.928391
## 10	0.2884284	8.319093
## 11	0.2630015	6.916981
## 12	0.1842748	3.395722

Second STPCA

##	length	%
## 1	0.2719225	7.394185
## 2	0.3133202	9.816954
## 3	0.1257545	1.581420
## 4	0.1611458	2.596797
## 5	0.2812371	7.909432
## 6	0.2727954	7.441730
## 7	0.2222145	4.937929
## 8	0.2200091	4.840402
## 9	0.3193051	10.195575
## 10	0.1230490	1.514106
## 11	0.2590368	6.710005
## 12	0.5921272	35.061464

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



Choropleth map



# Spatial weight matrix No 7; Rook Contiguity Weights

Eigenvalues of the matrix  $C$

##	[1]	40.532365	13.932358	6.927813	1.213339	0.186595	0.038757
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	0.000000	0.000000	-0.165735	-0.606031	-0.761634
##	[55]	-1.602721	-4.559724	-6.564000	-7.888383	-9.530608	-90.560798



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	64.50991	64.50991
## 2	22.17426	86.68416
## 3	11.02607	97.71023

Length of the subvectors:

First STPCA

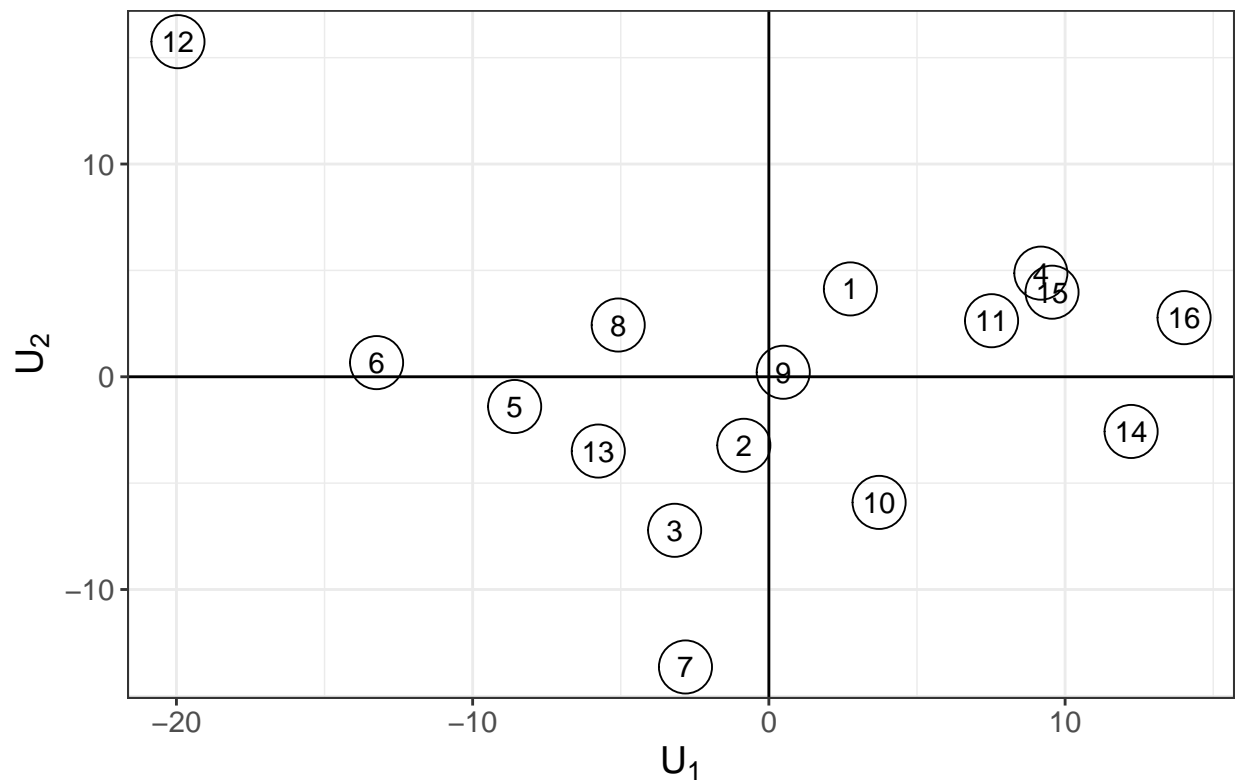
##	length	%
## 1	0.37616199	14.1497842
## 2	0.03824608	0.1462763
## 3	0.19329590	3.7363305
## 4	0.11575561	1.3399362
## 5	0.12953629	1.6779650
## 6	0.78637097	61.8379307
## 7	0.07681456	0.5900477
## 8	0.07844076	0.6152953
## 9	0.28403307	8.0674788
## 10	0.22481830	5.0543270
## 11	0.16007675	2.5624564
## 12	0.04713512	0.2221719



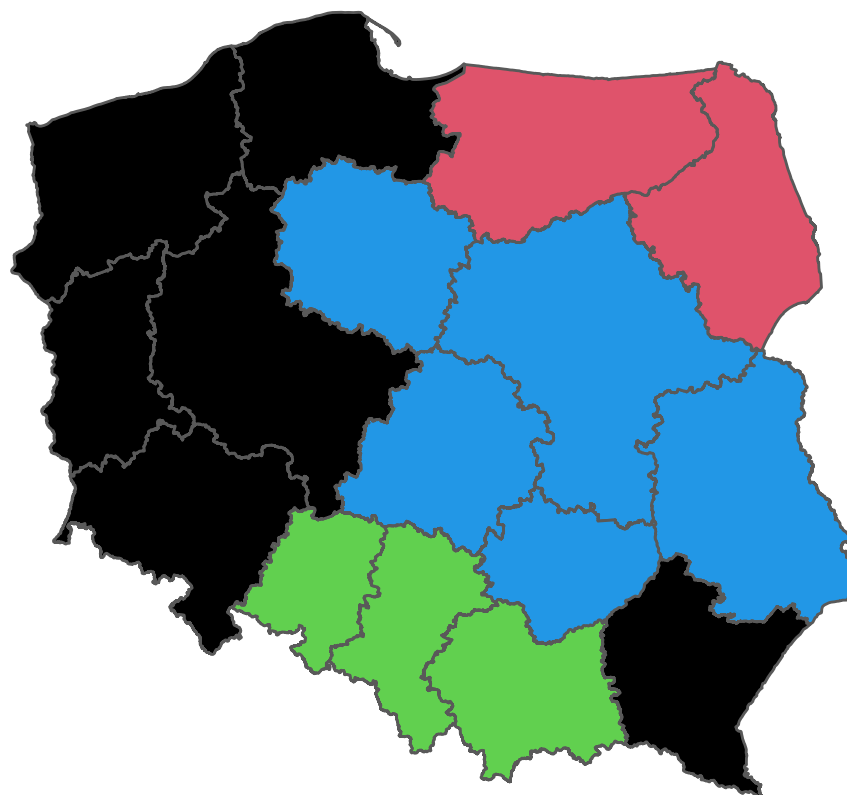
## Second STPCA

##	length	%
## 1	0.24368610	5.9382913
## 2	0.08395658	0.7048707
## 3	0.33167263	11.0006736
## 4	0.35199333	12.3899305
## 5	0.17307379	2.9954537
## 6	0.23043926	5.3102254
## 7	0.33025516	10.9068469
## 8	0.34020769	11.5741271
## 9	0.24742647	6.1219859
## 10	0.20152485	4.0612263
## 11	0.40787447	16.6361580
## 12	0.35157091	12.3602105

Projections of the 16 regions in a space of the first two positive spatio-temporal principal components



Choropleth map



Ujemne wartości własne macierzy C

Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	74.084644	74.08464
## 59	7.796659	81.88130
## 58	6.453212	88.33452

Length of the subvectors:

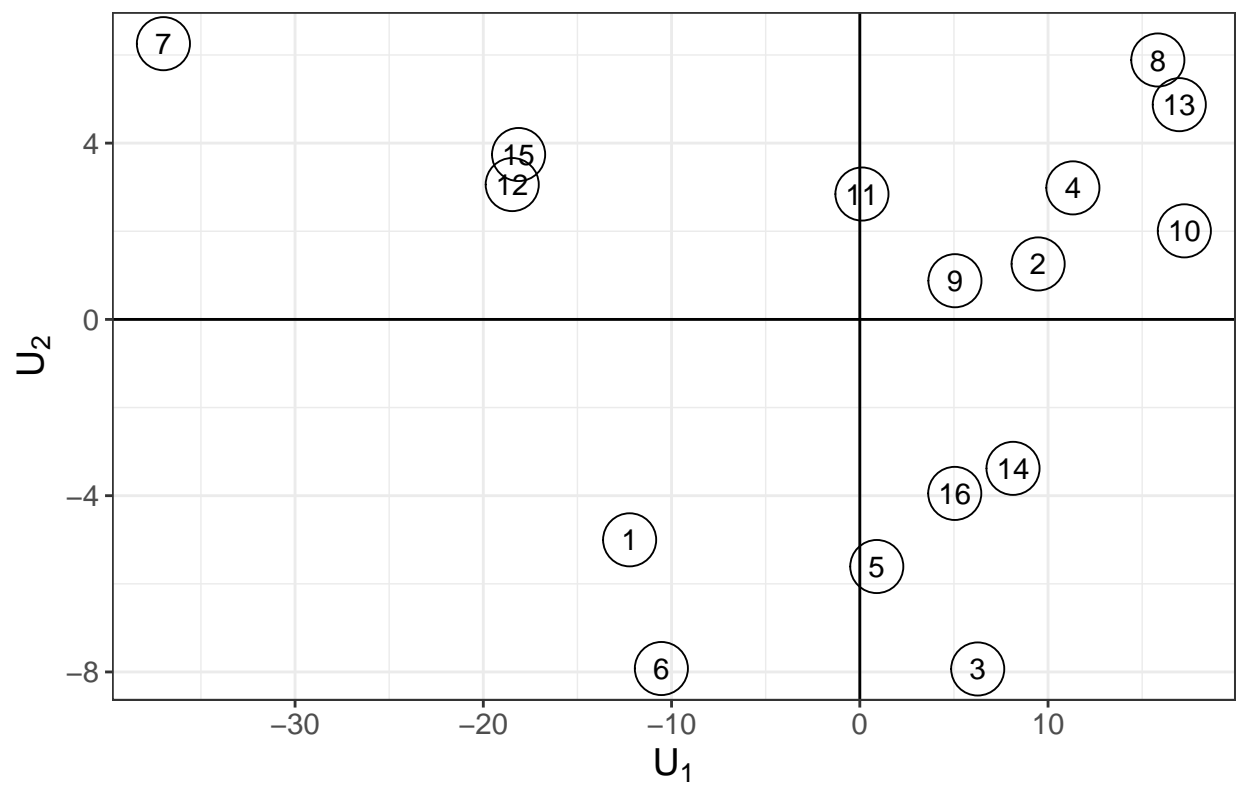
First STPCA

##	length	%
## 1	0.1660005	2.755617
## 2	0.3004880	9.029305
## 3	0.2685065	7.209577
## 4	0.3502702	12.268919
## 5	0.3447819	11.887455
## 6	0.1901257	3.614777
## 7	0.3809107	14.509294
## 8	0.3821196	14.601540
## 9	0.1674561	2.804156
## 10	0.2784231	7.751940
## 11	0.2896070	8.387220
## 12	0.2276005	5.180199

Second STPCA

##	length	%
## 1	0.06473106	0.419011
## 2	0.44982257	20.234034
## 3	0.41052609	16.853167
## 4	0.37699607	14.212603
## 5	0.30353764	9.213510
## 6	0.12793607	1.636764
## 7	0.20642275	4.261035
## 8	0.18956653	3.593547
## 9	0.32406055	10.501524
## 10	0.29780795	8.868957
## 11	0.25960840	6.739652
## 12	0.18617722	3.466196

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



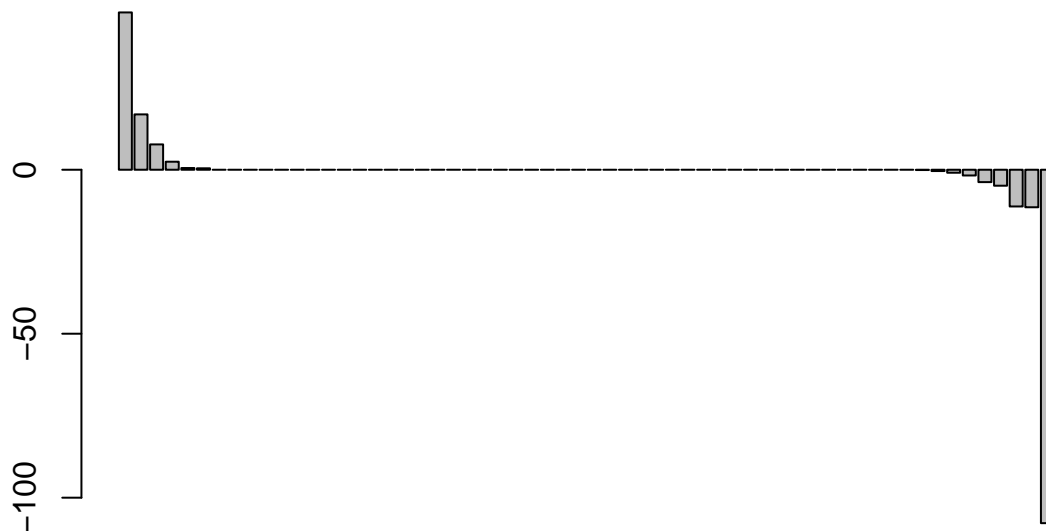
Choropleth map



#### Spatial weight matrix No 8; Shared-Boundary Weights

Eigenvalues of the matrix  $C$

##	[1]	47.963004	16.881587	7.721441	2.449181	0.518876	0.439847
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	0.000000	0.000000	-0.090352	-0.398916	-0.908449
##	[55]	-1.749259	-3.770056	-4.869001	-11.197918	-11.443413	-107.762116



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	63.13087	63.13087
## 2	22.22024	85.35110
## 3	10.16328	95.51438

Length of the subvectors:

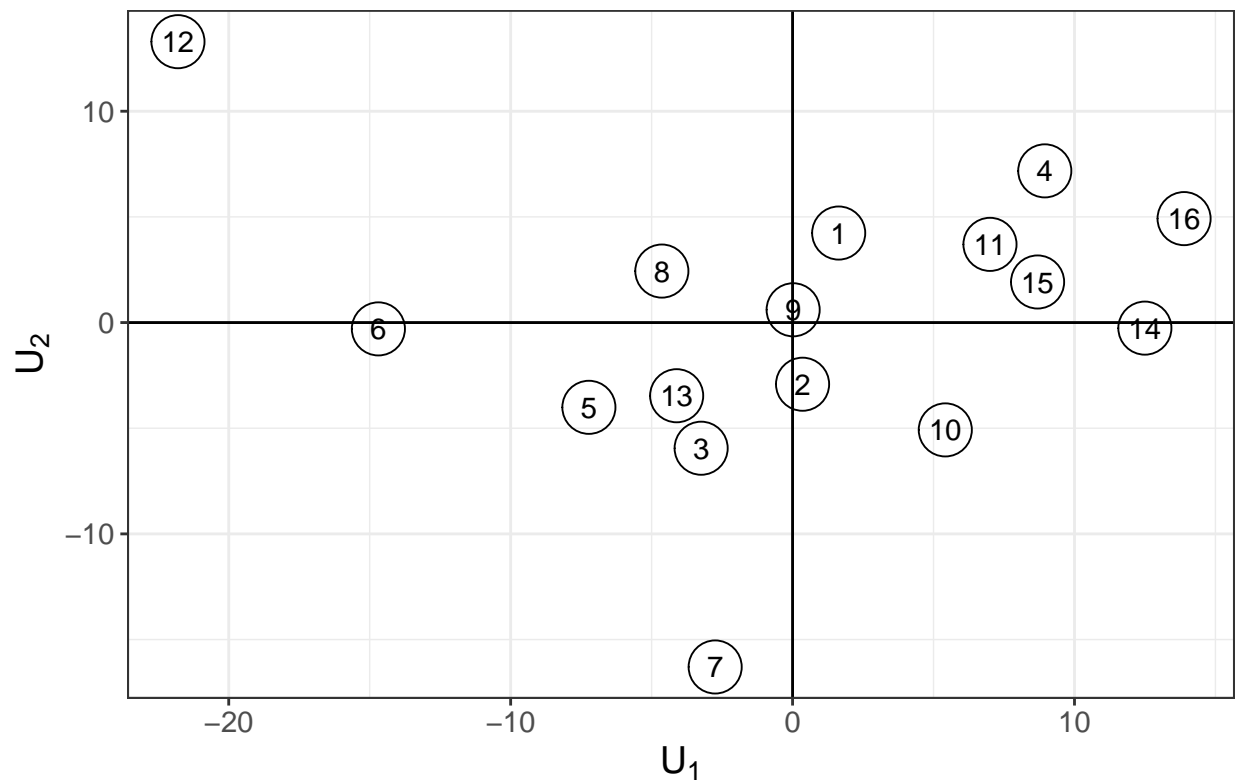
First STPCA

##	length	%
## 1	0.41592692	17.29952045
## 2	0.03674154	0.13499410
## 3	0.28869671	8.33457900
## 4	0.09854418	0.97109563
## 5	0.13393333	1.79381373
## 6	0.73194418	53.57422862
## 7	0.08120783	0.65947109
## 8	0.08445794	0.71331434
## 9	0.32560424	10.60181240
## 10	0.19141611	3.66401261
## 11	0.14698476	2.16045186
## 12	0.03044769	0.09270618

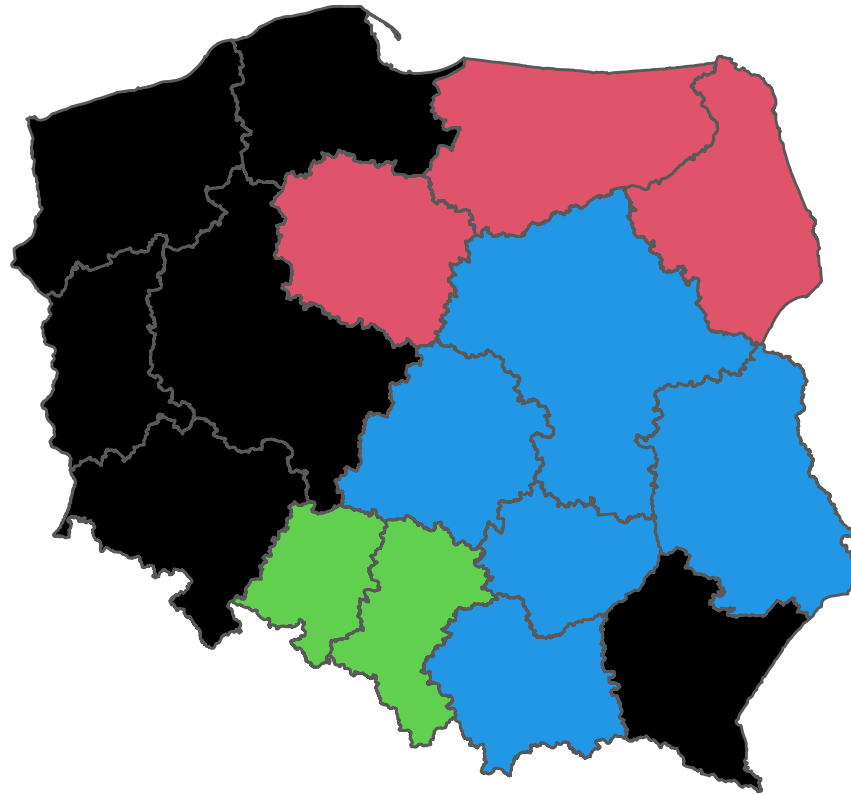
## Second STPCA

##		length	%
## 1	0.22736524	5.1694951	
## 2	0.08276449	0.6849962	
## 3	0.36864357	13.5898078	
## 4	0.30852119	9.5185324	
## 5	0.18899979	3.5720920	
## 6	0.34829204	12.1307342	
## 7	0.31499423	9.9221368	
## 8	0.32155760	10.3399290	
## 9	0.30225592	9.1358643	
## 10	0.09528256	0.9078766	
## 11	0.41124993	16.9126508	
## 12	0.28488392	8.1158847	

Projections of the 16 regions in a space of the first two positive spatio-temporal principal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	75.787686	75.78769
## 59	8.048003	83.83569
## 58	7.875349	91.71104

Length of the subvectors:

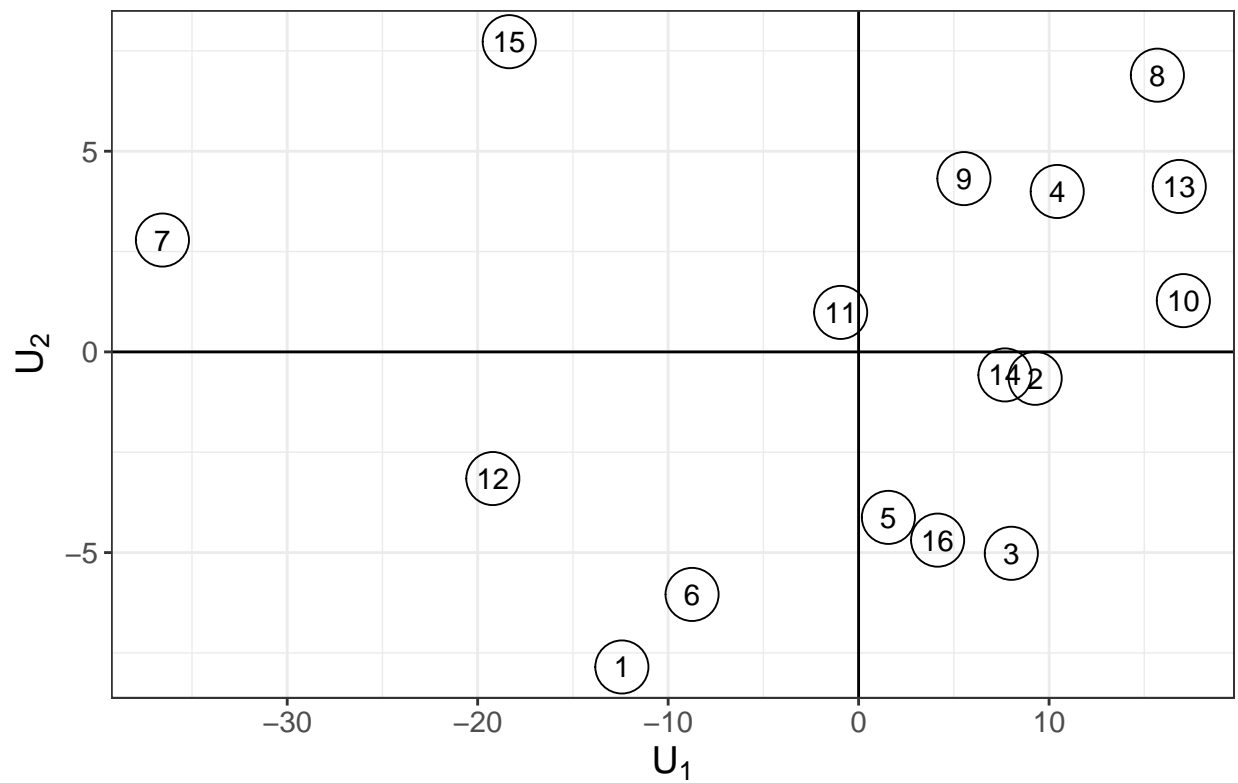
First STPCA

##	length	%
## 1	0.1744378	3.042855
## 2	0.2657787	7.063831
## 3	0.2191240	4.801534
## 4	0.4132690	17.079125
## 5	0.3376948	11.403776
## 6	0.2541858	6.461045
## 7	0.3798001	14.424810
## 8	0.3778442	14.276620
## 9	0.1729670	2.991759
## 10	0.2781622	7.737419
## 11	0.2524734	6.374280
## 12	0.2083974	4.342947

## Second STPCA

##		length	%
## 1	0.02690921	0.07241058	
## 2	0.51078890	26.09052986	
## 3	0.33299489	11.08855966	
## 4	0.28320139	8.02030275	
## 5	0.32357208	10.46988906	
## 6	0.18597598	3.45870650	
## 7	0.08682481	0.75385469	
## 8	0.09999111	0.99982212	
## 9	0.28103593	7.89811943	
## 10	0.30907551	9.55276688	
## 11	0.26686488	7.12168639	
## 12	0.38043859	14.47335207	

Projections of the 16 regions in a space of the first two negative spatio-temporal principal components



Choropleth map

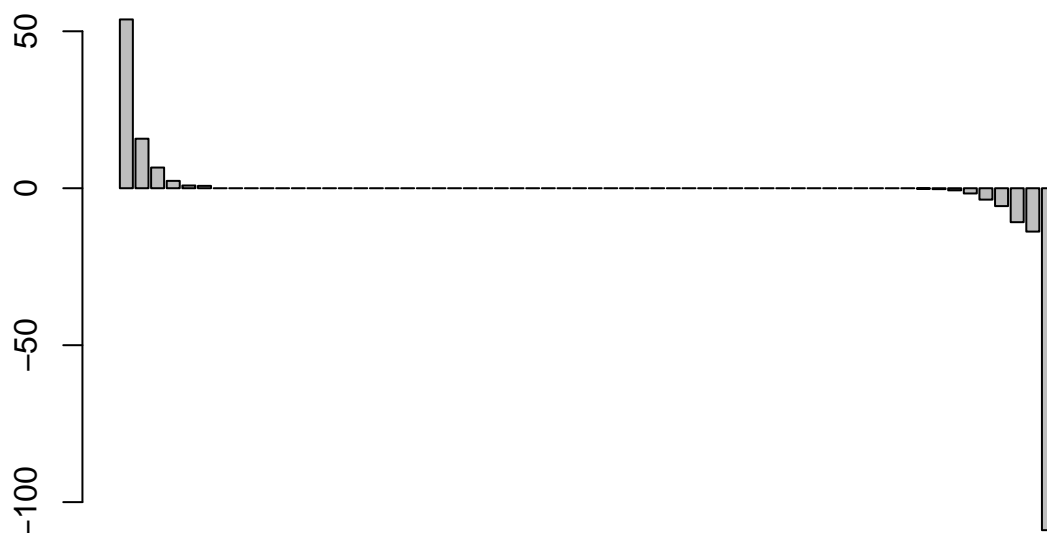




#### Spatial weight matrix No 9; Combined Distance–Boundary Weights

Eigenvalues of the matrix  $C$

##	[1]	53.774124	15.779350	6.592388	2.343596	0.877063	0.757386
##	[7]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[13]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[19]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[25]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[31]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[37]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[43]	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	[49]	0.000000	0.000000	0.000000	-0.176714	-0.288074	-0.675155
##	[55]	-1.666440	-3.628744	-5.691233	-10.813374	-13.813627	-108.927362



Positive spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 1	67.113706	67.11371
## 2	19.693685	86.80739
## 3	8.227741	95.03513

Length of the subvectors:

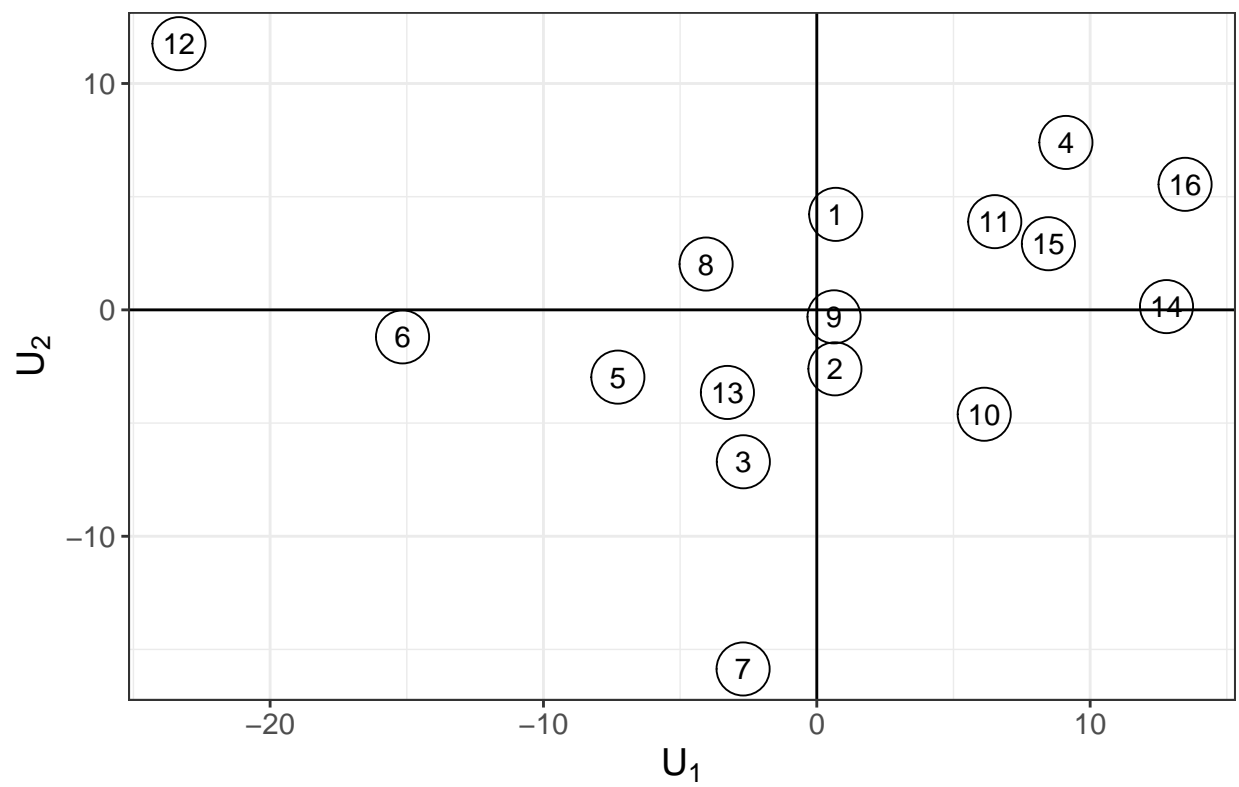
First STPCA

##	length	%
## 1	0.44117945	19.46393108
## 2	0.05973612	0.35684037
## 3	0.31112878	9.68011192
## 4	0.07649362	0.58512735
## 5	0.14616036	2.13628508
## 6	0.69123289	47.78029030
## 7	0.07142217	0.51011265
## 8	0.07632626	0.58256981
## 9	0.34018057	11.57228182
## 10	0.22324234	4.98371442
## 11	0.15087620	2.27636292
## 12	0.02690210	0.07237228

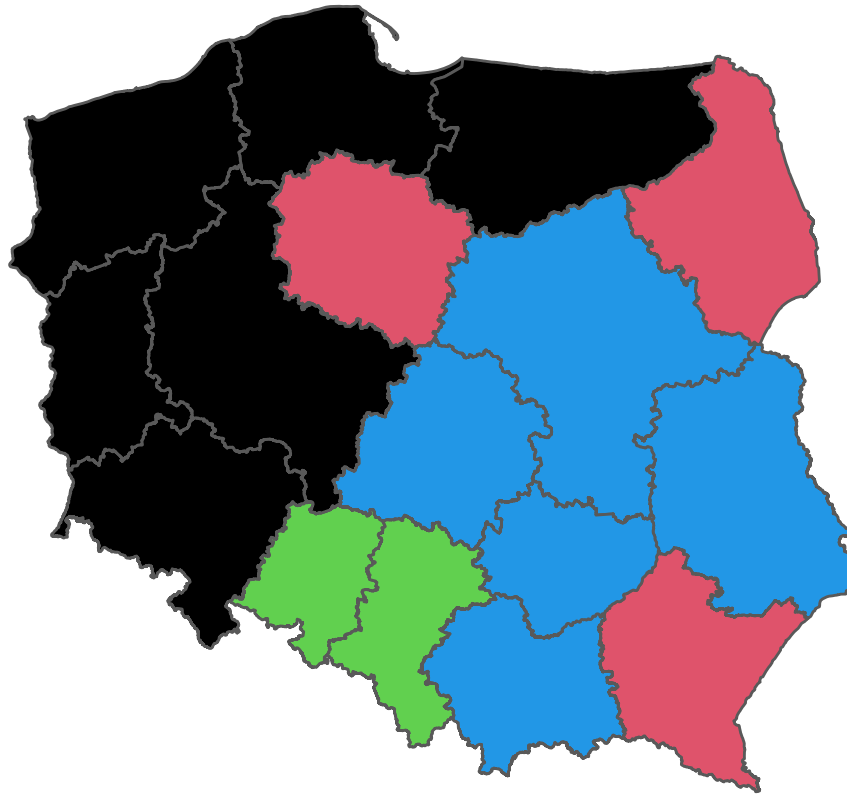
Second STPCA

##	length	%
## 1	0.18530878	3.4339343
## 2	0.06584066	0.4334992
## 3	0.34447562	11.8663450
## 4	0.33922013	11.5070299
## 5	0.16445554	2.7045624
## 6	0.36206499	13.1091058
## 7	0.35933208	12.9119541
## 8	0.36760769	13.5135413
## 9	0.23485532	5.5157024
## 10	0.13281806	1.7640637
## 11	0.40723203	16.5837929
## 12	0.25800134	6.6564690

Projections of the 16 regions in a space of the first two positive spatio-temporal princpal components



Choropleth map



Negative spatio-temporal principal components

Proportion of variability:

##	Proportion	Cumulative
## 60	74.771295	74.77129
## 59	9.482124	84.25342
## 58	7.422652	91.67607

Length of the subvectors:

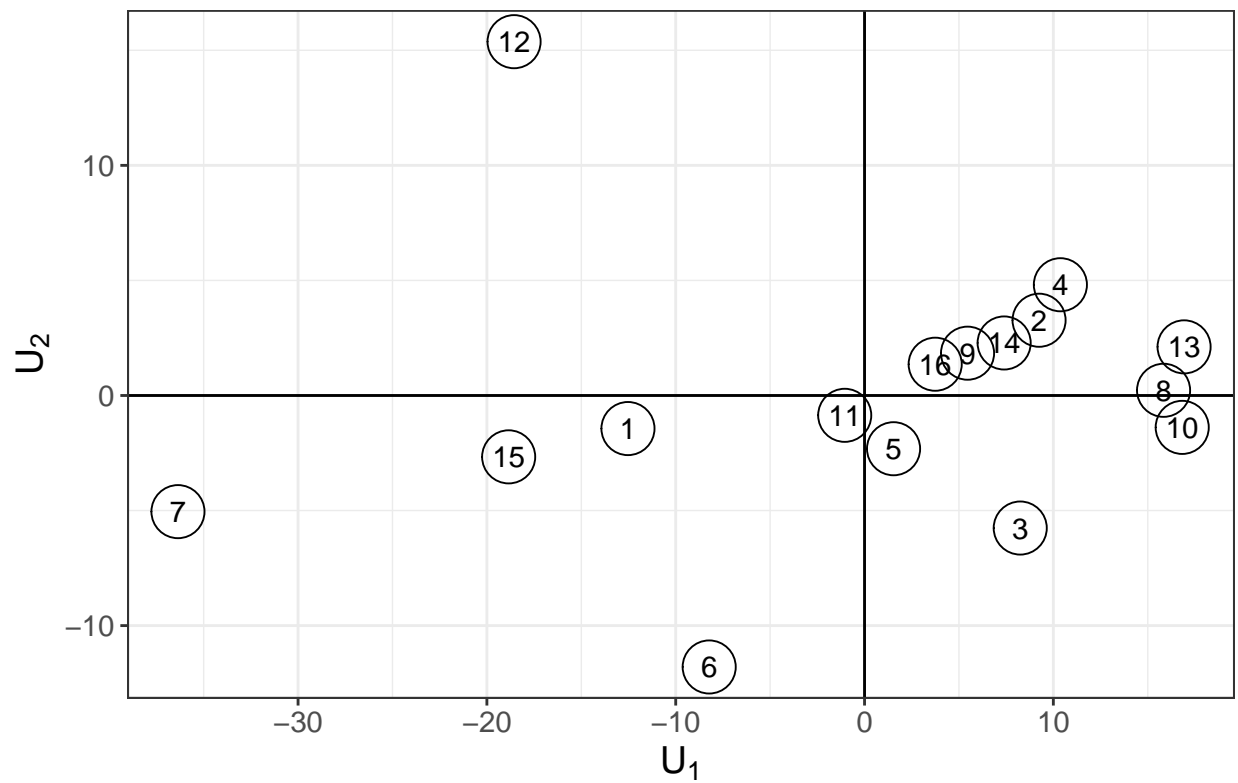
First STPCA

##	length	%
## 1	0.1598981	2.556739
## 2	0.2681946	7.192834
## 3	0.2015655	4.062867
## 4	0.4146731	17.195380
## 5	0.3364369	11.318978
## 6	0.2703091	7.306698
## 7	0.3735840	13.956497
## 8	0.3716115	13.809512
## 9	0.1752946	3.072820
## 10	0.2856079	8.157186
## 11	0.2534848	6.425456
## 12	0.2223743	4.945032

## Second STPCA

##		length	%
## 1	0.1782140	3.176021	
## 2	0.7096576	50.361387	
## 3	0.3196829	10.219713	
## 4	0.1502861	2.258590	
## 5	0.2619505	6.861806	
## 6	0.1650966	2.725690	
## 7	0.2709612	7.341999	
## 8	0.2673647	7.148390	
## 9	0.1297275	1.682922	
## 10	0.1870914	3.500321	
## 11	0.1418812	2.013027	
## 12	0.1646248	2.710133	

Projections of the 16 regions in a space of the first two negative spatio-temporal princpal components



Choropleth map

